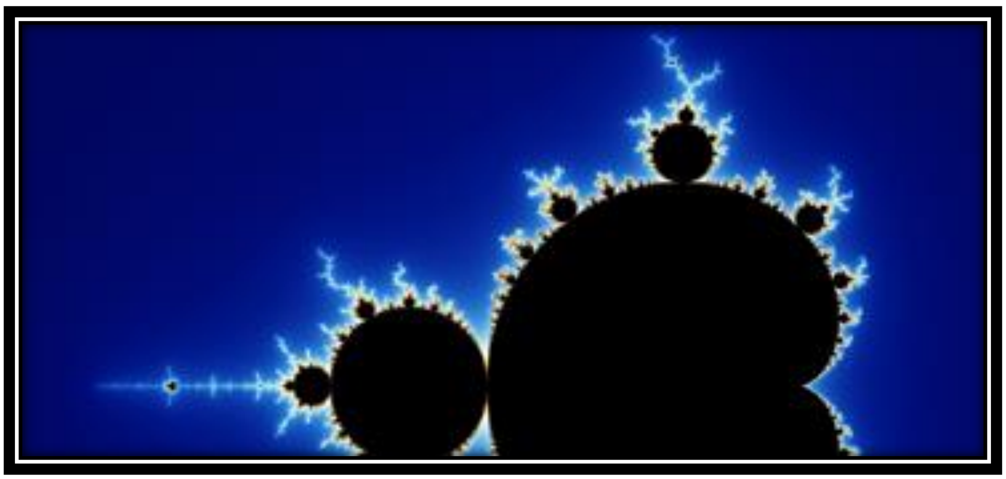
MATHEMAGIC CAMP

You'll discover how to twist arithmetic, outsmart logic puzzles, write unbreakable codes, dig into freaky fractals, build a geometric pantograph, step through a sheet of paper, multiply by drawing lines, perform math tricks that really look like magic, decode cryptograms, and memorize absolutely anything using a hundred–year-old secret code. You'll never be bored with math again.



Created by Aurora Lipper, Supercharged Science

www.SuperchargedScience.com

This curriculum is aligned with the National State Standards and STEM.

Lessons

Introduction: What is Math, and Why Bother with it?

Greetings, and welcome to the study of math. This unit was created by a mechanical engineer, university instructor, airplane pilot, astronomer, robot-builder and real rocket scientist … me! When I was in high school, I was so excited about math that I not only starting my own profitable math tutoring business, which was marketed mostly to my friends, but I also attended extra math classes in the evening at a local college where I was the only one with a curfew of 9 p.m. I have the happy opportunity to teach you everything I know about math over the next set of hands-on lessons – something that's not usually a part of math unless it's a game, and those get tiring after awhile, usually because they aren't related the real world.

I promise to give you my best stuff so you can take it and run with it … or fly!

When I first started out writing this study unit, I was amazed at how the math camps out there for the brainiacs who had *way* above-average math skills operated. There was even one camp where the kids were solving math problems that even adults didn't know how to solve. I also found a handful of remedial math camps for kids failing math during the academic year who needed to spend their summers making up for lost time. And those camps were overstuffed with busywork-style worksheets and endless rounds of drills, none of which were stimulating or designed to make kids think.

I found no place for kids in the middle of the spectrum, who want to continue honing their math skills in a fun and challenging way. That's really sad, because that's where most of the population is. There was absolutely nothing for the kids in the middle of the road, because the current math camp content out there was either designed for the super-smart kids or the ones falling way behind. That's when I decided to do something about it.

While we're not going to solve the world's current unsolvable math mysteries, or spend time with remedial work, what we *are* going to focus on is content that is fun, innovative, creative, and designed to make you think. And there's no silly cartoon-animations, fake problems designed to look real (how many pink elephants fit in the bathtub?), or endless rounds of busywork. You get enough of that elsewhere, so you won't find it here. What you will find is real math, just like real scientists use every day, so you know what to expect when you get out there in the real world. (Like I said: no pink elephants.)

So, that leads us to the first real question: *What is math?*

Math can be compared to a very useful tool, like a hammer, or a collection of tools like a set of screwdrivers. A lot of kids get frustrated and bored with math because many textbooks concentrate a lot on teaching the small, meticulous details of each and every type of tool. That's one of the fastest ways to kill your passion for something that could have otherwise been really useful!

Don't get me wrong – you do need to know how to tell a hammer from a screwdriver. But can you tell me *when* to use the hammer instead of the screwdriver? It's really important to focus on how and when to use the different tools. This is my practical approach to teaching the subject.

Most kids think math just means numbers, when the truth is that math is much more than just numbers and being good at multiplying! There are three main areas in math (at least when you first start out). Some kids enjoy adding and dividing, and for them, math is all about numbers. However, if you're really good with shapes and how they relate, then you might enjoy geometry. And if you are good at solving puzzles and people think you're unbeatable at certain games, chances are that logic will be a great match for your skills.

We're going to discover what math *really* is, and how we can use it in our everyday lives in a way that's really useful.

Mathemagic Camp is chock-full of demonstrations and hands-on activities for two big reasons. First, they're fun. But more importantly, the reason we do activities is to hone your math skills, which usually don't show up in the science arena until way later, like in high school or college. One of the biggest mistakes teachers make when teaching math is that they don't connect it back to the real world. They treat it like a bunch of problems on paper that need to be solved, as if the story ended there.

But you already know where math is around you: it's counting back change at the grocery store; it's figuring out how much fuel you need to make it to the next gas station; it's fitting all the boxes into the back of your truck; it's how to beat the kid down the street at chess. It's everywhere, if you only know where and how to look.

The skills in math take time and practice to master. But it's important not to get so lost in practice sessions that you lose sight of the goal. Imagine learning a new sport, and you were always practicing, practicing, practicing… and never got around to playing a game, never kept score, never heard the crowd cheer or felt the sense of pride that comes from scoring a goal. You actually need both: practice *and* performance, and most teachers only settle for practice and forgo the real reason you need to learn this stuff in the first place: to be able to handle the science side of things later on down the road.

One thing I didn't realize about math (but you probably already know) was that it's not always right. What I mean is that sometimes math gives deceptive answers that have to be interpreted for them to make any sense. For example, I remember one time I was doing a calculation and my answer to the problem actually resulted in *two* answers. One answer was 53 feet, and the other was 6 inches. Both answers solved the set of equations I was trying to solve. Now, how did I know which one was really right? I mean 6 inches is different from 53 feet. I had to look back at the original problem, and as soon as I did, I realized that the answer was 6 inches, since there was no way I was going to find a 53-foot badger.

Ideally, you'd learn both science and math skills together in tandem, so you could see how one affected the other; how one was used in a way that made the other possible; where math leaves off and where science takes over, and how they intertwine. For example, you could really see how to model a car on paper as a mass-spring-damper system and write an equation to describe the motion the driver feels while driving down the road, solve that differential equation, plot out the solution on a graph, find the points where your solution exists, and run back and put a big X on the car where you want the wheels in order for the ride to be as smooth as possible for the rider (turns out that those points are at the *center of percussion*, kind of like the "sweet spot" on a bat when a baseball batter hits the ball in exactly the right spot so there's almost no force felt at the grip). But it's not always possible to teach math this way because not every teacher will have these skills.

To sum it up: math is a tool that helps scientists model the real world down on paper so we can make the problems easier to solve. Once we solve them, we have to bring them back to the real world by making sense of what we did on paper. That's usually where we find out how good of a job we did in the first place. That's why I always do my math problems in pencil, and I write everything down so I can easily find my mistakes.

Math by itself is an art, but math combined with science is pure joy and fulfillment, the kind I want to share with you. I'm going to give you a lot of different activities to help you develop your math techniques in learning how to think. Good luck with Mathemagic Camp!

For the Parent/Teacher:
Educational Goals for Math Camp

This unit is broken into several sections, each of which contain easy, fun activities to get your feet wet and your mind thinking. You'll find labs in arithmetic, geometry, cryptography, logic, probability, statistics, lightning calculations, and more. If you feel like you're getting a little lost, skip over to the simpler activities and just have fun.

By the end of the labs in this unit, students will be able to:

- Demonstrate how to add, subtract, multiply, and square numbers quickly in their heads
- Be able to memorize anything by utilizing the phonetic code
- Be able to test any number for single-digit divisibility quickly and easily
- Perform math magic tricks to amaze and amuse, and add to their weekly allowance (if they charge admission)
- Figure out what day of the week a date falls on in any century of the Gregorian calendar
- Design and build a pantograph copy machine, just like the ones used before Xerox came along
- Be a whiz at figuring out sizes, shapes, and geometric patterns in a snap
- Learn how to develop a fractal and why they are so useful
- Understand how to multiply using graphical placeholders instead of numbers
- Create their own fun by playing some of the most popular (and oldest) mathematical games of all time, including those developed by ancient Chinese mathematicians, Benjamin Franklin, and more
- Wrestle with logic puzzles and paradoxes and still be able to find their way home
- Understand how quiz shows use probability and statistics
- Crack ciphers and create unbreakable secret codes that not even a computer whiz can hack into
- Follow a set of written instructions for mathematical investigation

This course isn't designed to take the place of a math curriculum, but add to it in a fun and meaningful way that will jump-start your students' interest in math long-term. By using the activities in this unit, you'll be able to bring math to life so it's no longer flat boring and meaningless to students. Enjoy!

Master Materials List for All Labs

This is a list of the materials that you will need to do *all* of the activities, experiments and projects in this curriculum program. The set of materials listed below is just for one lab group. If you have a class of 10 lab groups, you'll need to get 10 sets of the materials listed below. For 10 lab groups, an easy way to keep track of your materials is to give each group a number from 1 to 10, and make up 10 separate lab kits using small plastic tubs or baskets. Put one number on each item and fill each tub with the materials listed below. Label the tubs with the section name, like *Mathemagic Study Kit,* and you will have an easy way to keep track of the materials and build accountability into the program for the kids. Copy these lists and stick them in the bin for easy tracking. Feel free to reuse items between lessons and unit sections. Most materials are reusable year after year.

- brass fasteners, 4
- cardboard (scrap piece) or wood, or other old table space to practice on (your table may get scratched)
- color pens or markers, 3 different colors
- cups, 3
- dice (two 6 sided and two 12 sided)
- dollar bill
- drill with drill bits
- masking tape
- mechanical pencils, 2
- paper
- paperclips, 11
- pen
- pencil
- scissors
- scissors or small hand saw to cut the yardsticks into three 16″ lengths and one 8″ length
- toilet paper tube
- yardsticks or metersticks, 2

Lesson #1: The Magic of 11's

Overview: Here's our first lesson. It is so easy that one night, I wound up showing it to everyone in the pizza restaurant. Well, everyone who would listen, anyway. We were scribbling down the answers right on the pizza boxes with such excitement that I couldn't help it – I started laughing right out loud about how excited everyone was about math … especially on a Saturday night.

When you do this calculation in front of friends or family, it's more impressive if you hand a calculator out first to an unsuspecting friend, letting them know that you are "testing to see if the calculator is working right," Ask for a two-digit number and have them check the calculator's answer against yours.

Materials

- Pencil
- Paper

Activity
We're going to be able to multiply any two digit number by 11 magically in our head. At first, you'll want to use a paper and pencil, but you'll soon feel confident enough to do the entire calculation completely in your head. Here's the deal:

Let's figure out this one: 11 x 23 = ?

Take the 2 and 3 and spread them apart, so it looks like this: 2☐3
(That little box means "space", where you'll be placing a digit in a moment.)

Now add 2 + 3. Did you get 5?

Put the "5" in the box to get your answer: 11 x 23 = <u>253</u>!

That's it! How cool is that?

Let's try another one: 11 x 45 = ?

First spread apart the 4 and 5, then add 4 + 5 to get 9.

Now put 9 between 4 and 5 to get 11 x 45 =<u>495</u>!

How about this one: 11 x 86 = ?

Spread apart the 8 and 6 like this: 8☐6

Now add 8 + 6 to get 14. But wait a second… is the answer 8146? That doesn't sound right, because the answer has to be a three-digit number, not four! So here's how to handle it: place the 4 in the box, and carry the 1 to the 8 and add it to make 9.

So 11 x 86 = <u>946</u>!

What do you think is going to happen once you show this to your friends? If they're like *my* friends, then they're going to ask you to do the biggest two-digit number you can think of. So let's so that one right now:

What is 11 x 99?

The first step looks like this: 9☐9

Since 9 + 9 = 18, write the 8 in the box and carry the "1" and add it to the first 9 to get 10.

Your final answer is 11 x 99 = <u>1,089</u>!

Let's try one last example: What is 11 x 78?

The first step looks like this: 7☐8

Add 7 + 8 to get 15, and write the "5" in the box. Where does that "1" go from the 15? Add it to 7 to get 8.

11 x 78 = <u>858</u>!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 11 x 11
2. 11 x 27
3. 11 x 43
4. 11 x 49
5. 11 x 50
6. 11 x 67
7. 11 x 79
8. 11 x 89
9. 11 x 92
10. 11 x 96

Lesson #2: Multiply a 3-Digit Number by 11

Overview: If you liked multiplying two-digit numbers by 11, chances are you're curious about what to do with a three-digit number, like 213. It's only one more step to figuring out the answer, making this trick equally as impressive as the two-digit version.

Materials

- Pencil
- Paper

Activity

If you haven't yet tried multiplying 11 by a two-digit number, stop and do that now. This lesson is a whole lot easier if you are already comfortable doing the two-digit multiplication of 11. Let's try a three-digit multiplication of 11.

What is 11 x 213?

Before we start, how many digits do you expect your answer to have? If you pretend the 11 is a 10 and multiply the 213 by 10, we get a four-digit number. That tells us that our answer must have four digits.

Your first task is to space the first and last digits apart, this time with two spaces between them like this: 2 ☐☐ 3

Now we tackle this problem the same way we did the two-digit multiplication of 11's. The digit for the right box is found by adding the tens and ones together: 1 + 3 = 4. Now we have: 2 ☐4 3

The digit for the left box is found by adding the hundreds and the tens together: 2 + 1 = 3.

We get: 11 x 213 = <u>2,343</u>

Let's try another! Can you figure out 11 x 124 = ?

First, spread apart the first and last digit, and add your two boxes like this: 1☐☐4

The digit for the right box is found by adding the tens and ones digits together: 2 + 4 = 6 which makes it: 1☐64

You can figure out the digit for the left box by adding the digits from the hundreds and the tens together like this: 1 + 2 = 3.

Now we get the final answer of: 11 x 124 = <u>1364</u>

Do you think you can figure out 11 x 444 without writing anything down? Try it now before turning the page.

If we were doing this on paper, first we'd write out 4□□4

The right box's digit is found by adding the tens and ones digits together: 4 + 4 = 8 to make 4□84

The digit in the left box is found by adding the hundreds and tens together: 4 + 4 = 8

Did you get 11 x 444 = <u>4884</u>? Great!

Let's try a slightly harder one: 11 x 456 = ?

First, write the first and last digit out like this: 4□□ 6

The digit in the right box is found by adding the tens and ones digits together, which is: 5 + 6 = 11. But oh, no! We can't put two digits in a box, remember? So what can we do?

Simple! Place the ones digit (1) in the box and carry the tens digit (which also happens to be a 1) one place up, like this: 4□16

The left box's digit is found by adding not only the hundreds and tens together, but also that carried over 1 to get: 4 + 5 + 1 = 10. Uh-oh! Another two-digit answer.

So we have to carry the 1 one spot to the left and leave the zero in the left box. Adding the 1 to the hundreds digit gives 4 + 1 = 5, so your final answer is:

Therefore, 11 x 456 =<u>5,016</u> (Whew!)

Now let's figure this one out: 11 x 789 = ?

First write 7□□ 9

The digit for the right box is found by adding 8 + 9 = 17. Place the 7 in the right box and carry 1 to the left box. Just put a little tick mark above the box so you can remember it's there.

The left box's digit is found by adding 7 + 8 + 1 (carried forward) = 16. Put the 6 in the left box.

Add the 1 to the 7 to get 8.

Final answer: 11 x 789 = <u>8,679</u> *Ta-daa!*

Of course, once the word gets out that you are an absolute genius when it comes to multiplying three-digit numbers by 11, some big kid is going to challenge you with this one, so why not do it now?

See if you can figure out 11 x 999 before turning the page…

Did you get 10,989?

Here are the steps in case you need them:

First write (or think): 9☐☐ 9

Now the digit for the right box is found by adding the ones and tens digits together: 9 + 9 = 18. Write the 8 in the right box and carry the one 1 to the left box: 9☐ 89

The digit for the left box is found by adding together the hundreds and tens together (which happens to all be the same number in this problem, but try to keep it straight): 9 + 9 = 18 + 1 (carried forward) = 19. Write 9 in the left box.

The 1 is carried over to the leftmost digit (in the thousands place) and added to the 9 to get 10.

11 x 999 = 10,989!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 11 x 163
2. 11 x 235
3. 11 x 345
4. 11 x 479
5. 11 x 659
6. 11 x 748
7. 11 x 997
8. 11 x 982
9. 11 x 873
10. 11 x 769

Lesson #3: Multiply by 12's

Overview: Do you think you'll need to know how to multiply by 12 or 11 more? Think of it this way: How often do you need to figure out how many dozen you need of something? It comes up a lot more than needing to know how many batches of 11, doesn't it? That's because of the way we've decided to group things mathematically as a society.

Here's why: We picked 12 based on how we used to count on our fingers using the "finger segment" system. If you look at your hands, you'll notice that your index finger has three segments to it. So do your middle finger, ring finger, and pinkie. Since you have four fingers, you actually have 12 sections for counting with (we're not including your thumb, which is the pointer… your thumb rests on the section you're currently on). When your thumb touches the tip of your index finger, that means "1." When your thumb touches the middle segment, that's "2," and the base segment is "3." The tip of your middle finger is "4," and so on. That's how we came to use the 12-in-a-batch system.

If you're wondering why we didn't use the 24-in-a-batch system (because you have two hands), that's because one hand was for 1-12 and the second hand indicated the number of batches of 12. So if your left hand has your thumb on the ring finger's base segment (9) and your right hand has the thumb touching the index finger's middle segment (2 complete batches of 12, or 2 x 12), the number you counted to is: 24 + 9 = 33.

Fortunately we now have calculators and a base-10 system, so this whole thing worked out well. But still the number 12 persists! So this is a very useful skill to have at your fingertips. It's very similar to the shortcut used when multiplying by 11, but it also involves some doubling. You'll find this is a really cool (and FAST) way to multiply by 12. And it's a lot faster than using the Babylonian finger-segment system. Try some problems on your own and check your work with a calculator.

Materials

- Pencil
- Paper

Activity: If you skipped over multiplying by 11's, stop and do it now. You'll find learning how to multiply by 12 builds on how you multiply by 11, and makes this lesson a whole lot easier to master.

Let's figure out 564 x 12 =?

First, how many digits do you expect your answer to be? If you pretend the 12 is a 10, do you see how you get a four-digit answer? It's important to have an idea about how big of a number your answer should be before you start, so you can check or adjust your final answer before you finish.

To do this trick, we need the 564 to be a four-digit number, not a three-digit number (you'll see why in a moment). So put a zero at the front of the number, making it 0564.

Starting at the right side of the number, we double the 4: (4 x 2) = 8. This is the ones digit for your answer.

8

Now double 6 and add the 4 from the ones place to your answer: (6 x 2) + 4 = 16. The 6 is the tens digit for your answer. We will carry the 1 from the 16 to the hundreds place.

68

Your next step is to double the 5 and add 6. Don't forget to also add the 1 that carried over to get: (5 x 2) + 6 + 1 = 17. Write the 7 for the hundreds digit and carry the 1 to the next digit over.

768

Now you'll see why we needed the zero at the front of the number. Double the 0 (that's easy), and add 5 and 1(carried from the previous step): (0 x 2) + 5 + 1 = 6. This is the thousands digit. And we are also happy our answer has four digits!

6,768

564 x 12 = <u>6,768</u>

If you're wondering if it might have been easier to just do the multiplication in the traditional way, remember that this is the first time you're doing this, so you can expect it to take longer than usual. With a little practice, you'll be able to multiply these numbers quickly in your head. When you get really good at this, you can do it starting at the front of the number because you'll be able to see when and if there are any "carries" in the problem.

Let's try another one so you can get good at this trick. 382 x 12 =?

How many digits do you expect to have?

If you said four, you're right!

So first, put zero at the front: 0382

The first calculation you need to do is double the 2 to get the ones place digit: (2 x 2) = 4

4

Next, double the tens digit and add the 8 add 2: (8 x 2) + 2 = 18. Write the 8 for the tens digit.

84

Now double the hundreds digit 3 and add 8. Don't forget the carried 1. (3 x 2) + 8 + 1 = 15. Write 5 for the hundreds digit and carry the 1.

584

Finally, double the 0 and add 3 and the carried 1 to get: (0 x 2) + 3 + 1 = 4

4,584

382 x 12 = <u>4,584</u>

Now, it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 11 x 543
2. 12 x 45
3. 12 x 326
4. 12 x 769
5. 12 x 1345
6. 12 x 3461
7. 12 x 7532
8. 12 x 8989
9. 12 x 9999
10. 12 x 98749

Lesson #4: Divisibility

Overview: Do you know that 8 can be divided by 2 completely? The answer may be yes because it is a small number; therefore division can be carried out easily. However, can you figure out if 9,054,137,828 can be divided by 7 completely without a remainder?

Conducting long division for large numbers can be tedious (and boring). This lesson gives us an opportunity to answer such questions within seconds.

Divisibility is a concept which implies that a number should be divided by another completely. Since 8 can be divided completely by 2, we say it is divisible by 2. In this lesson, we will discuss quick tests that can make us say whether a number is divisible by 2, 3, 4, 5 and 7.

Materials

- Pencil
- Paper

Activity

Divisibility test for 2

Let's start with an easy divisibility test: testing for the divisibility of 2. A number is divisible by two if the last digit is divisible by 2. Numbers that are divisible by two are 0, 2, 4, 6, and 8. If the last digit of your number is one of these, then the entire number is divisible by 2. Examples are 36, 92, 50, 18, and 46.

Circle the numbers that are divisible by 2: 24 90 67 76 6 49 26 32 313 358 927 1024 512

Divisibility test for 3

This is an interesting test, because it seems mysterious how it works. To find out if a number is divisible by 3, we first *add* the digits of the number together and of the sum is divisible by 3, then the original number itself is also divisible by 3.

Let's try an example: 144

First, we'll figure out the sum of the digits: 1 + 4 + 4 = 9.

Does 3 go into 9? You don't have to figure out how many times it does – it's just a yes or no thing.

Since the answer is *yes*, then 144 is divisible by 3!

Try this one before turning the page: Is 273,645 divisible by 3?

First, you need to sum the digits: 2 + 7 + 3 + 6 + 4 + 5 = 27

Does 3 go into 27? Yes!

So you've just figured out that 273,645 is divisible by 3.

Divisibility test for 4

A number is divisible by 4 if the last two digits are divisible by 4 (or if they are two zeros). For example, 2,016 is divisible by 4 since 16 is divisible by 4.

Is 244 divisible by 4?

The last two digits (2<u>44</u>) form the number 44. When we look at the 44, we immediately know that 4 goes into 44, so that means that 244 is divisible by 4.

Is 18,336 divisible by 4?

Check out the last two digits. They form the number 36, and since 36 is divisible by 4 (again, it doesn't matter right now how many times 4 goes into 36), the number 18,336 is divisible by 4.

What about a million? Is 1,000,000 divisible by 4?

Yes, because the last two digits are "00" so automatically the number is divisible by 4.

Divisibility test for 5

This is probably the easiest of all divisibility tests. Simply look at the last digit on the number. If you find a 0 or 5 in the ones place, then the number is divisible by 5. Can you easily see that these numbers: 2<u>5</u>, 24,51<u>0</u>, 6,53<u>5</u> and 2,382,639,403,482,51<u>0</u> are divisible by 5?

What about these? 522,33<u>4</u>, 524,25<u>1</u> and 63,734,63<u>8</u> ? (These are not divisible by 5.)

Divisibility test for 6

This one looks difficult, but it's really a combination of two previous tests we've just learned.

Remember that 6 = 3 x 2, which means that to get to 6, we must first multiply 3 and 2 together.

What this really means is that a number is divisible by 6 if it is divisible by *both* 2 and 3.

For example, is the number 1,134 divisible by 6? Let's find out.

First, we can test for the divisibility of 2. Since the last digit is a 4, this number is divisible by 2. So far, so good. But to be divisible by 6, it's also got to pass one more test.

Now let's test for divisibility by 3. We'll sum the digits 1 + 1 + 3 + 4 = 9. Since 9 is divisible by 3, this number passed the second test.

The result? 1134 can be divided by 3 *and* 2, so it's also divisible by 6.

Divisibility test for 7

There are a couple of ways to check for divisibility by 7. This one happens to be one of my favorites, and it makes use of a strange 3-2-1 pattern. Let's do an example so you see how it works:

Our job is to figure out if the number 6, 124 314 is divisible by 7.

Since the number has 7 digits, we write a pattern of 3, 2 and 1 beginning with 3 as shown in the left column of numbers. Next assign each number with a + or - sign, beginning with positive downward but shifting from positive to negative or negative to positive each time you hit a 1.

Now write the multiply symbol (x) after the 3-2-1 pattern of numbers, and then write your original number *backwards* so that the ones place gets multiplied by the +3, the tens place digit gets multiplied by the +2, and so forth. See if you can figure out what I've done here:

+3	+3 x 4	+3 x 4 = 12
+2	+2 x 1	+2 x 1 = 2
-1	-1 x 3	-1 x 3 =-3
-3	-3 x 4	-3 x 4 = -12
-2	-2 x 2	-2 x 2 = -4
+1	+1 x 1	+1 x 1=-1
+3	+3 x 6	+3 x 6 = 18

Now add up all the numbers together in the last column to get 14 (12 + 2 + (-3) + (-12) + (-4) + (-1) + 18 = 14). Since the number 14 is divisible by 7, then the number 6,124,314 is divisible by 7.

This is a tricky divisibility test, so let's try another example: Is 18, 102 divisible by 7? Try it in the space below before turning the page for the steps.

The number 18, 102 has 5 digits, which means we will have a 3-2-1 pattern made up of 5 digits.

+3 x 2 = 6

+2 x 0 = 0

-1 x 1 =-1

-3 x 8 = -24

-2 x 1 = -2

Sum up the numbers on the right side gives -21. Disregard the + or – sign and notice that 21 is divisible by 7. So the number 18,102 is divisible by 7!

What about other numbers? You can tell if a number is divisible by 9 if it's divisible by the numbers that make up 9 (9 = 3 x 3). So 270 is divisible by 9 since it's also divisible by 3. The number 100 is divisible by 10 (10 = 2 x 5) since it's also divisible by 2 and 5.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. Identify the number(s) that are divisible by 2 in the following list.
   301, 3645, 3673
2. Identify the numbers that are divisible by 3 in the following list.
   3981, 430, 4598, 72624
3. Which one of the following numbers is divisible by 7
   5894, 56723, 17259
4. Is 2353740 divisible by 4?
5. Which one of the following numbers is divisible by both 2 and 5?
   10002, 453970, 637385
6. A number is divisible of 2 and 3, will it be divisible by 5?
7. If a number is divisible by 2 only, will it be divisible by 4?
8. Which of the following numbers is divisible by 3?
   45769, 25784, 2391
9. List any three 3-digit numbers that are divisible by 5.
10. List two 4-digit numbers that are divisible by 4.

Lesson #5: On What Day Were You Born?

Overview: People celebrate their birth dates every year, but are these days similar to the *days* they were born? Not usually! This is not only a neat trick but a very practical skill – you can figure out the day of the week of anyone's birthday.

If you were born in the 20th Century, (1900-1999), we can use math to find out on which day of the week you were born. If you're a little too young for this, try it with a parent or grandparent's birthday when you first start out. (I'll show you how to adjust it for different centuries a little later.)

Materials

- Pencil
- Paper

Table of Months

Jan. 1 (0 for leap year)	
Feb. 4 (3 for leap year)	
Mar. 4	Aug. 3
Apr. 0	Sept. 6
May 2	Oct. 1
June 5	Nov. 4
July 0	Dec. 6

Activity:

Let's figure out what day of the week May 3rd, 1917 is. We need to figure out four special numbers, which we will call A, B, C, and D, in order to answer this question.

First, let's make the letter A equal to the number formed by the last two digits of the year (19<u>17</u>). So A = 17 for our example.

Now make B the number of times 4 divides A (ignore the remainder). So 17/4 =4 remainder 1. Ignoring the remainder makes B = 4.

C will be the month you were born. This can be found in the Table of Months. Since May is 2, then C = 2.

D is the day of the month you were born, from the date May 3, 1917 makes D = 3.

Table of Days

Sun 1	Thu 5
Mon 2	Fri 6
Tue 3	Sat 0
Wed 4	

Now we add together all the numbers to get:
A + B + C + D = 17 + 4 + 2 + 3 = 26

We divide the sum (26) by 7 and figure out the remainder. 26/7 = 3 with a remainder of 5.

Now use the Table of Days to figure out the day being determined using the remainder (5). So May 3rd , 1917 is on a Thursday!

Note: the codes shown here are special just to the years in the 1900s. If you'd like to be able to expand this to other centuries, you'll need to use the codes listed below and learn how to shift them.

What about years 2000-2099?

The general formula is: Month Code + Date + Year Code – (the biggest multiple of seven)

For example, for Nov. 18, 2006: 2 + 18 + 0 = 20.

20 – (7 * 2) = 20 / 14 = 6 so Nov 18, 2006 is on a Saturday using the codes below!

(Do you notice how the following codes are different than they were for 1900s?)

Day Codes:

- Sunday = 0
- Monday = 1
- Tuesday = 2
- Wednesday = 3
- Thursday = 4
- Friday = 5
- Saturday = 6
- Sunday = 0 or 7

Month Codes:

- January = 6*
- February = 2*
- March = 2
- April = 5
- May = 0
- June = 3
- July = 5
- August = 1
- September = 4
- October = 6
- November = 2
- December = 4

*For leap years (2000, 20004, 2008, 2012, 2016…)
the code for Jan = 5 and Feb = 1.

Year codes:

- 2000 = 0
- 2001 = 1
- 2002 = 2
- 2003 = 3
- 2004 = 5
- 2005 = 6
- 2006 = 0
- 2007 = 1
- 2008 = 3
- 2009 = 4
- 2010 = 5
- 2011 = 6
- 2012 = 1
- 2013 = 2
- 2014 = 3
- 2015 = 4
- 2016 = 6
- 2017 = 0
- 2018 = 1
- 2019 = 2
- 2020 = 4
- 2021 = 5
- 2022 = 6
- 2023 = 0
- 2024 = 2
- 2025 = 3

We don't have to memorize 2000-2099 because we know how to divide numbers since the table repeats itself.

Here's how it works: if you need the code for 2061, divide 61 by 4 to get 15 (with a remainder of 1 that we ignore), so 2061 has a year code of 61 + 15 = 76. Don't forget to subtract out any multiple of 7, so we get 76 – 70 = 6. The year code for 2061 is 6!

This works the same way for the 1900s: For Dec. 3, 1998 we have 98 divided by 4 which gives 24 (with a remainder of 2 that we ignore), so 1998 has a year code of 98 + 24 + 1 = 123. Now subtract out the biggest multiple of seven (which is 119) to get 123 – 119 = 4. 1998 has a year code of 4!

What about year codes for other centuries?

Did you notice how I added "1″ to the year code in the previous example? That's because I had to shift it over since it's in the 1900s. For the 1800s we'd shift it by 3. Let me show you how:

Abraham Lincoln's birthday is Feb 12, 1809. 2009 has a year code of 4, that we need to add 3 to (this is the shift by 3), so we get 7 (which reduces down to zero). The year code for 1809 is 0.

So, his birthday is: 2 + 12 + 0 – 14 – (biggest multiple of seven) = 14 – 14 = 0.

Lincoln was born on a Sunday!

For 2100 dates, you'd need to add 5 to the year code (or subtract 2 from the year code). For example, 2109 has a year code of 4 + 5 = 9. Subtract out 7 gives a year code of 2. For 1700s, you'll treat them just like the 2100s.

Why does this work?

We're using a Gregorian calendar. While this type of calendar was created in 1580s, it wasn't until Wed, Sept 2, 1752 when it was adopted by England and American colonies. 1752 has a year code of zero, which is why this method won't work for any dates before this, as they were on the Julian calendar. Note that the Gregorian calendar repeats itself every 400 years, so you can convert any future date into a date near 2000. For example, March 19, 2361 and March 19, 2761 will both be on a Sunday.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

Identify the days corresponding to the following dates given in the format,: mm/dd/yyyy

1. 11/16/1997
2. 1/1/1997
3. 05/27/1995
4. 08/15/1997
5. 07/15/1977
6. 03/01/1977
7. 11/24/1974
8. 06/27/1958
9. In a certain family, Janet was born on May 20, 1992 while Lewis was born on March 31, 1996. Who was born on Sunday?
10. On which day is April 30, 1960?

Lesson #6: $1 Word Search

Overview: Have you ever heard of a dollar word search? It's a special kind of puzzle where the letters in a word add up to a coin value. For example, an **A** is worth a penny, the letter **B** is worth two cents, **C** is worth three cents, and so on. A word like "excellent" is worth $1.

Materials

- Pencil
- Paper

Activity

First, we need to assign each letter of the alphabet an amount. You start with the first letter, A, and that's worth 1 cent. B is worth 2 cents. C is worth 3 cents… and on up to Z which is worth 26 cents.

Now can you tell me how it's possible for gold to be worth less than silver?

Let's find out. Gold = G-O-L-D. This means, G=7, O=15, L=12 and D=4, so GOLD = 7 + 15 + 12 + 4 = 38 cents. But how much is silver worth?

The letters of silver are: S=19, I=9, L=12, V=22, E=5 and R=18, so SILVER = 19 + 9 + 12 + 22 + 5 + 18 = 85 cents. Hence silver is worth more than gold!

But the real point of the game is to find as many words as you can that equal exactly $1. I'll start you out with a couple so you can see how this works: excellent and discipline are worth $1. So are first names like Suzanne, Kristin, Henrietta, and Christian. Several animal names in the zoo are dollar-worthy, as are a couple of undersea creatures. A certain number (when spelled out) which is less than 100 is worth a dollar. So is a flushing device in the bathroom. Can you figure it out?

I have more than 1,700 words in my $1 word search… can you find out a few?

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. What is the word "bucket" worth?
2. Determine the monetary value of the word "toilet."
3. What is a "starfish" worth?
4. The shortest sentence in English is "Go." How much is it equivalent to?

Which one of the following has the greatest monetary value?

5. Supper, dinner, lunch
6. Monday, Sunday, Tuesday

Lesson #7: Isn't That SUM-thing?

Overview: This is a neat trick that you can use to really puzzle your friends and family. You're going to sum five different numbers; three of them are random picks from the audience. The trick is that by only knowing the first number, you can predict the final number every time.

Materials

- Pencil
- Paper

Activity: Do this trick by yourself first before doing it in front of an audience. Let's pretend you already have, and that you're doing it live in front of people. You're going to magically add five numbers (a couple of which you don't even know yet) and come up with the answer *before* you know all the numbers in the sum. Here's what you do:

645

Ask your audience to give you any three-digit number. Say for example the kid in back shouts out "645!" Write this number at the top of your list.

Now you're going to predict the final end number before knowing the rest of the numbers. On a scrap of paper, write down this number: 2643, and hand it to someone for safe-keeping until the end of your trick.

How did I know to write 2643? Simple! From the original number: 645, put a "2" in front of the entire number (in the thousands place), and subtract 2 from the last (the ones) digit. Once this number is safely hidden away, ask for another three-digit number.

645
473

Suppose the kid in front calls out "473!". Write this new number below 645.

Now it's your turn to come up with a number. You can't just come up with any number though, because it's got to be done in a special way for this trick to work. Your new number is such that when you add it to the second number listed it gives 999.

645
473
526

So look at 473: what number do you need to add to 4 in the hundreds place to make it a 9? 5!

What number do you need to add to the 7 in the tens place to make it a 9? 2!

And what number do you need to add to the ones digit to make it a 9? 3! So your new number that you write below is 526.

645
473
526
128
+ 871
2,643

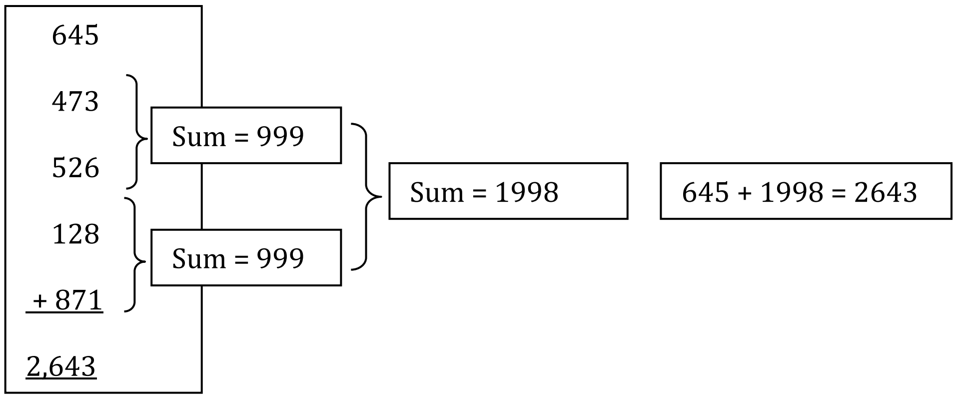
Ask your audience for another three-digit number. Suppose grandpa shouts out "128!" Write this under 526.

You need to do the 999 trick again (like you did to get 526). So quickly write 871 below 128.

Now ask for a volunteer to use a calculator to sum all five numbers and give you the answer. As they are punching in the numbers, quickly write down your initial guess of 2,643.

Now ask the person holding the scrap of paper to read the number. *Ta-daa!*

Were you able to figure this trick out? The real trick is that you're simply adding 1,998 to the first number that you received. No matter which other two numbers are given to you, make sure the number you write down makes each pair of numbers' sum 999. If you do this correctly twice, then the row of numbers you add together will be the same as the initial number plus 1,998.



Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

Predict the end result for the following numbers:

1. 235
2. 988
3. 002
4. 999
5. 427
6. 777
7. 559
8. What would be the difference between a number 769 and its predicted result based on the above knowledge?
9. A mathematician was given a number x and gave its end result as 2877. What is the value of x?
10. Suzanne was asked by her friend to predict the end results of 932 within a few seconds. If she was given 432 as one of the two additional numbers to complete the proof of the predicted number, which number did she write immediately afterwards?

Lesson #8: How to Add and Multiply Fast

Overview: Want a peek under the "hood" of my brain when I do a mental math calculation? This lesson includes steps that take a slow-motion, step-by-step snapshot of what goes on when I add numbers in my head. The first thing you need to learn is how to look at the *whole* problem when adding, and also learning how to multiply from LEFT to RIGHT, both of which are opposite from most math techniques out there. I'll show you how to do this. It's easy, and essential to working bigger numbers in your head.

Materials

- Pencil
- Paper

Activity

Let's take a look at the following problem: 123 + 372 = ?

First think about the hundreds place. I know this isn't the way most folks teach, but as I mentioned, try learning to add from left to right, because it's a lot faster to do in your head. So the first step is to get a general idea of the answer before even calculating it. For this problem, it looks like I am going to add about 100 (rounding down from 123) to 400 (rounding up) to get roughly 500.

Now look for patterns and shortcuts. I can add 25's a lot easier than oddball numbers like 123 and 372. So I adjust the math problem a little like this: 123 is close to 125, while 372 is close to 375. If we add these two, we get: 125 + 375 = 500. So we can expect the answer to be a little less than 500.

How much did I add to 123 to get it to 125? Just 2.

How much did I add to 372 to get it to 375? Just 3.

Add these two together to get 2 + 3 = 5. Subtract 5 from 500 to get the answer: 495.

Here's another problem: 576 + 93 = ?

93 is really close to 100. In fact, it's only 7 less, so I already see something I can use. Since it's easier to add 100 than 93, I can solve the problem like this: 576+100 – 7 = 676 – 7 = 669, which is a lot easier to do than the original program.

The answer is: 576 + 93 = 669

What about 432 + 79 = ?

Take a look at the tens place: there's a 3 and a 7, which sums to 10, so we can break 79 into 70 + 9. So the problem changes a bit: 432 + 79 becomes 432 + 70 + 9 = 502 + 9 = 511.

Now let's shift over to multiplication. This is where you'll learn how to multiply from left to right. Are you ready?

We're going to do a really simple one to get started: 300 x 126 = ?

Look at it like this: 300 = 3 x 100. First multiply 126 by 3 then multiply the result by 100.

Don't forget that 126 = 100 + 20 + 6. So now the problem becomes:

3 x 126 = (3 x 100) + (3 x 20) + (3 x 6)

3 x 100 = 300

3 x 20 = 60

3 x 6 = 18

Add all the numbers together to get 378. Multiply this by 100 by placing two zeros at the end so the final answer is: 37,800.

300 x 126 = <u>37,800</u>

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 23 + 74
2. 48 + 169
3. 627 + 192
4. 799 + 5692
5. 562 + 658
6. 20 x 236
7. 400 x 41
8. 300 x 344
9. 50 x 239
10. 203 x 456

Lesson #9: Squaring a Two-Digit Number

Overview: This neat little trick shortcuts the multiplication process by breaking it into easy chunks that your brain can handle. The first thing you need to do is multiply the digits together, then double that result and add a zero, and then square each digit separately, and finally add up the results.

Materials

- Pencil
- Paper

Activity: Finding the square of a number simply means multiplying the number by itself. For example, the square of 5 = $5^2 = 5 x 5 = 25$

What about larger numbers? What is: 23^2 = ?

This is done in a couple of steps. First, multiply the digits of the number (2 and 3) together to get 2 x 3 = 6. Now double this to get 6 x 2= 12, and add a zero to the end to get 120. Keep this number in mind.

Next, square the tens digit (2) to get 4. We want this number to be a two-digit number for this technique to work, so write 04.

Now square the ones digit to get 09.

Place these two numbers together to get 0409, and add the 120 to it to get the final result: 0409 + 120 = 529.

23^2 = 529!

Next example: 45^2 = ?

First, multiply 4 x 5 = 20, double it to get 40, and add a zero to get 400.

Now square the tens digit to get $4^2 = 16$.

Square the ones digit to get $5^2 = 25$.

Place these two numbers together to get 1625, and add 400 to get the final answer of 2,025.

$45^2 = 2025$!

How about a larger number? Find the square of 86. Try this on your own before turning the page for the solution.

First multiply 8 by 6 to get 48, double the product 48 x 2 = 96 and add a zero to get 960.

Square the tens digit to get $8^2 = 64$, and the ones digit $6^2 = 36$ and place them together to get 6,436.

Add 6,436 + 960 together using our addition trick we learned about recently. Since 960 is 40 away from 1,000, the problem becomes: 6,436 + 960 = 6,436 + 1,000 – 40 = 7,396.

$86^2 = 7{,}396$!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 21^2
2. 43^2
3. 37^2
4. 69^2
5. 99^2
6. 82^2
7. 58^2
8. 64^2
9. 53^2
10. 86^2

Lesson #10: How to Square Bigger Numbers

Overview: Squaring three-digit numbers is one of the most impressive mental math calculations, and it doesn't take a whole lot of effort after you've mastered two-digit numbers. It's like the difference between juggling three balls and five balls. Most folks (with a bit of practice) can juggle three balls. Five objects, however, is a whole other story (and *WOW* factor).

Once you get the hang of squaring two-digit numbers, three-digit numbers aren't so hard, but you have to keep track as you go along. Don't get discouraged if you feel a little lost. It's just like anything you try for the first time. When you're new at something, in the beginning you aren't very good at it. But with practice, these steps will become second nature and you'll be able to impress your friends, relatives, *and* math teachers.

Materials

- Pencil
- Paper

Activity: Make sure you're comfortable with squaring two-digit numbers before attempting this!

We're going to use a nifty little formula that works well for mental calculations.

$$A^2 = (A + d)(A - d) + d^2$$

What is 49^2 = ?

We're going to find an easier number that's close to 49 to work with. How about 50?

So *A* = the original number, and *d* is the distance from the original number to the "easier" number.

For our example, d = 50 – 49 = 1.

(A + d) = 50

(A – d) = 48

d^2 = 1

So now the problem becomes: 50 x 48 + 1

Let's work with this a little: What is 5 x 48? Multiply from left to right to get 240. (5 x 4 = 20, and 5 x 8 = 40, and when you add a zero to the end of the 20 (because it's in the tens place), you get the easy problem of 200 + 40 = 240.)

It's a simple step to finish the problem: 5 x 48 = 240, but remember it was 50 x 48, so add a zero to 240 to get 2,400. The last step is to add the d^2 term to get $49^2 = 2{,}401$.

Try solving this one before turning the page: $86^2 = ?$

I picked the "easy" number to be 90 (although you could have also picked 100). So for me, d = 4.

Using the formula, we have:

(A + d) = (86 + 4) = 90

(A - d) = (86 – 4) = 82

d^2 = 16

Multiply 90 x 82 like this: Start with 9 x 82 = (9 x 8) x 10 + 18 =720 + 18 = 738.

Adjust it so that it's 90 x 82 = 7,380 and add the d^2 = 16 term to get the final answer: 7,396.

$86^2 = 7{,}396$!

What is $186^2 = ?$

There are two good choices for "easy" numbers to choose: 200 and 100. I'll pick 100 (both will give you the same answer).

If we use 100, then d = distance between the two numbers: 186 – 100 = 86.

(A + d) = (186 + 86) = 272 (use the addition trick we've covered in a previous lesson)

(A - d) = (186 – 86) = 100

d^2 = 86^2, which we already figured out to be 7,396.

Now we'll figure out 272 x 100 = 27,200.

If we hadn't already known the solution to 86^2, you can do it quickly now using the simpler version of squaring two-digit numbers.

$186^2 = 27{,}200 + 7{,}396 = 34{,}596$

$186^2 = 34{,}596$!

What is 936^2 =?

Try to work this out in the space below before turning the page!

I'm going to pick the "easy" number be 900, so d = 36.

What is 36^2? Well, remember you can multiply the 3 x 6 = 18, double it to get 36, and add a zero for 360. Now work with squaring the 3 to get 09, and the 6 to get 36 and smoosh them together to get 0936. Add 0936 to 360 to get 36^2 = 1,296.

(A + d) = (936 + 36) = 972

(A – d) = (936 – 36) = 900

d^2 = 1,296

Now multiply 900 x 972 like this: 9 x 972 =? (Do your multiplication from left to right!) 9 x 972 = 8,748. Add two zeros to the end, since we're multiplying by 900 and not just 9 to get: 874,800 and add the d^2 = 1,296 term.

Before you add the d^2 = 1,296 term, look at it carefully. Do you notice how 1,296 is really close to 1,300?

So try adding it like this:

874,800 + 1,300 - 4 = 876,096

Therefore $936^2 = 876{,}096$!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 93^2
2. 193^2
3. 979^2
4. 249^2
5. 415^2
6. 84^2
7. 573^2
8. 333^2
9. 757^2
10. 696^2

Lesson #11: Folding a Cube

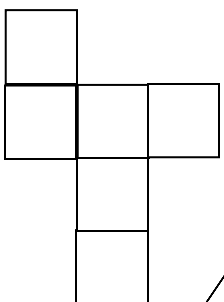
Overview: There's more than one way to fold a cube from a flat sheet of paper! In the video that goes with this lesson, I've used Post-It notes since they are square, but you can cut out your own pieces of paper and stick them together with tape.

Materials

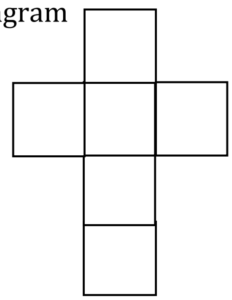
- Pencil
- Paper

Activity: Do you think that any of these three shapes can be folded to make a cube? Look carefully!

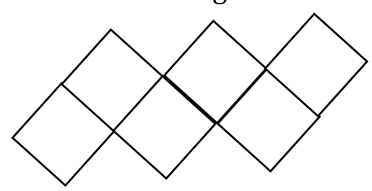
1st Diagram

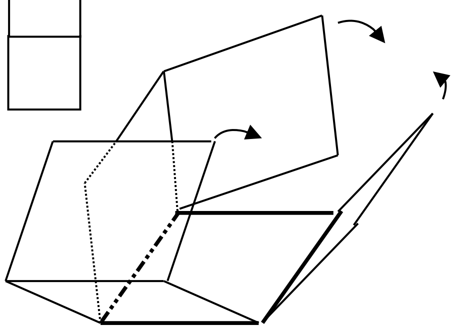


2nd Diagram

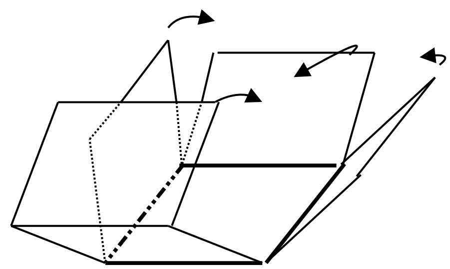


3rd Diagram

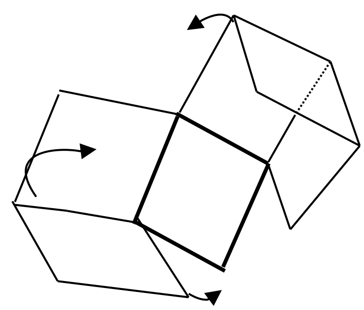




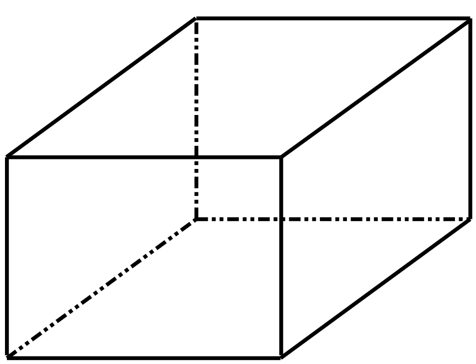
1st Diagram



2nd Diagram



3rd Diagram

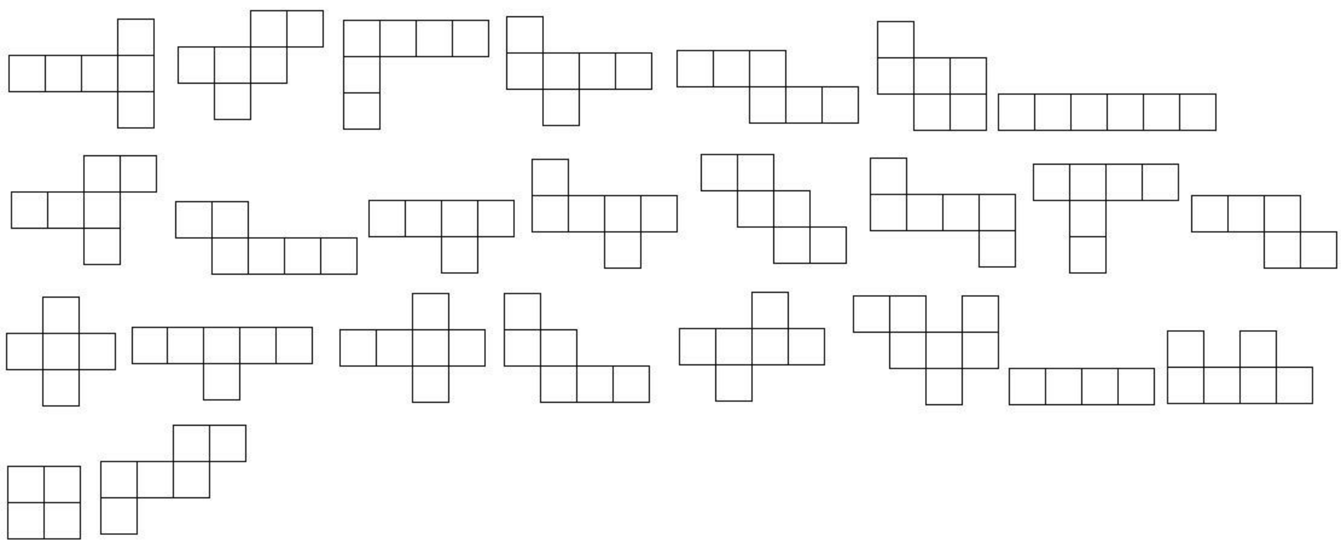


Final diagram

The first, second and third diagrams are folded as shown above to make the final diagram, a complete cube.

Exercises

1. How would you describe a cube?
2. How many faces does a cube have?
3. Which of the following diagrams can be folded into a cube? Circle the ones you think will make a cube! (Answers are found at the back of this book.)



Lesson #12: Möbius strip

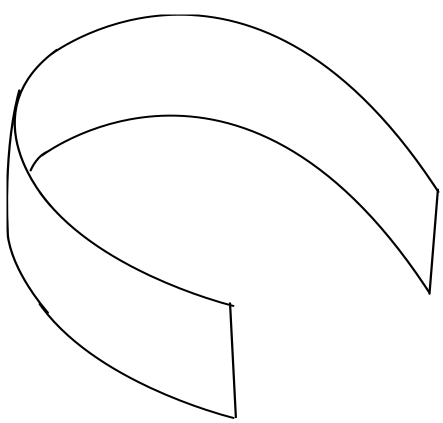
Overview: Although the Möbius strip is named for German mathematician August Möbius, it was co-discovered independently by Johann Benedict Listing, a completely different German mathematician, but at around the same time in 1858. Weird, right? But that's not the only strange thing about the Möbius strip. It's a non-orientable surface. This means it has a path that will take a traveler back to their point of origin. Are you completely confused now?

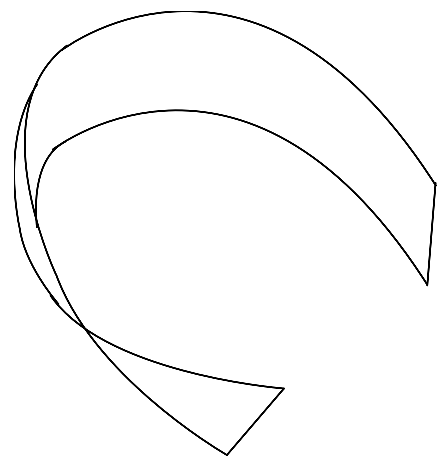
Materials

- Pen
- A pair of scissors
- Paper
- Tape

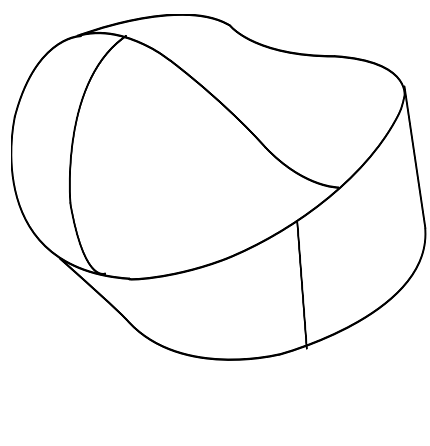
Activity: Let's make one of these mysterious Möbius trips and then we'll play with it!

First, cut out a strip of paper. Before taping it together to make a bracelet, twist one end a half turn like this:





Now connect the ends to form a Möbius strip.

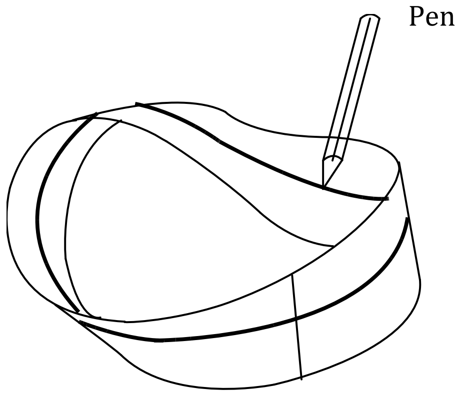


The strip has one side (well, two if you also count the thickness of the paper, but for now you can ignore this since it's so much smaller than the width),and unlike a circular strip which has two sides: the inside face and the outside face.

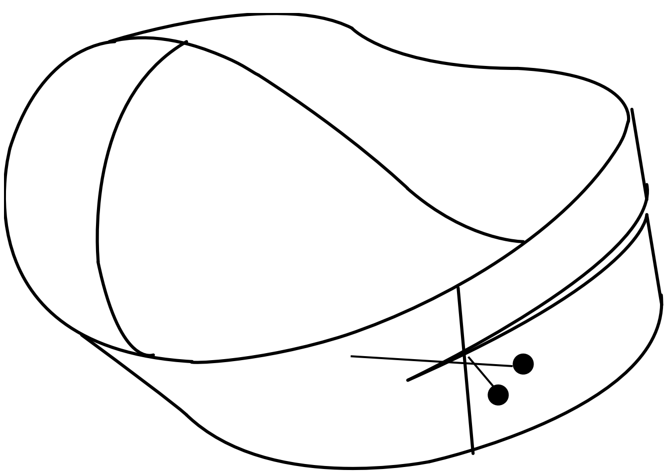
This is where the paradox is. A *paradox* is something that seems to contradict itself. It sort of makes sense, but then again it doesn't *really* make sense. It seems odd that a paradox fits into a math lesson, yet they do exist in mathematics. The Möbius strip is a fun example of a paradox that you can actually touch.

Here's the paradox: take a pencil and start drawing a line along the Möbius strip, and you'll notice that the line gets drawn on both the outside and the inside face, but you haven't lifted your pencil! This line turns out to be just *one* line.

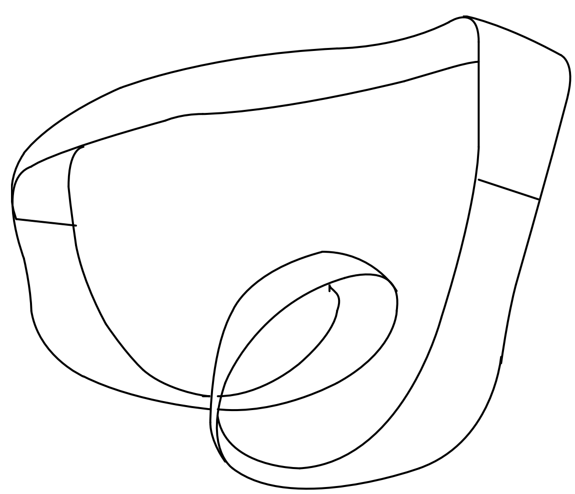
This shows that an object moving along the surface of the Möbius strip will trace one path that appears on its outside and inside face. I once made a slot car race track using flexible track to show kids how the car continuously runs on the track in a circle, but sometimes it's on the inside surface and sometimes it's on the outside!



What happens if we cut the strip in half longways? Grab your scissors and cut it just like it's shown in the diagram:



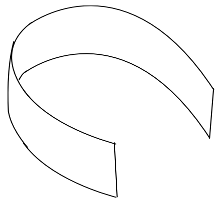
Did you get this shape? A strip with one twist?



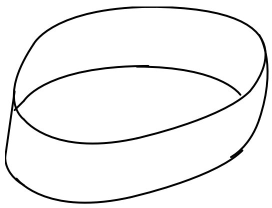
Exercises

Identify the figures shown below:

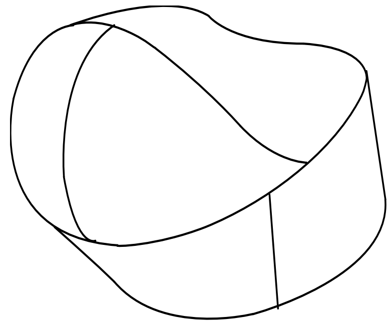
1.



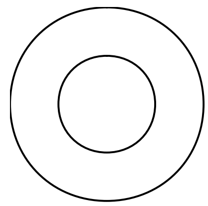
2.



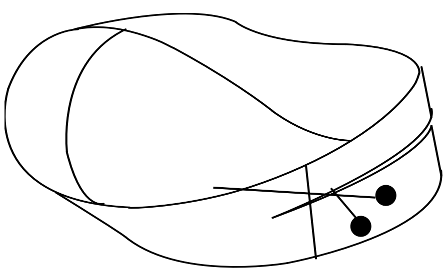
3.



4.



5. How many faces does a Möbius strip have?
6. How many edges does a Möbius strip have?
7. Given a strip of paper, at which angle should one rotate one of its ends before connecting to form a Möbius strip?
8. When a person cuts along the length of the Möbius strip to come up with a longer one, how many twists does he observe in the final strip?
9. Say you want to get two Möbius strips from an existing one. How would you do it?



10. If you cut the Möbius strip along its length as shown in 9 above, how many connected strips would you get?

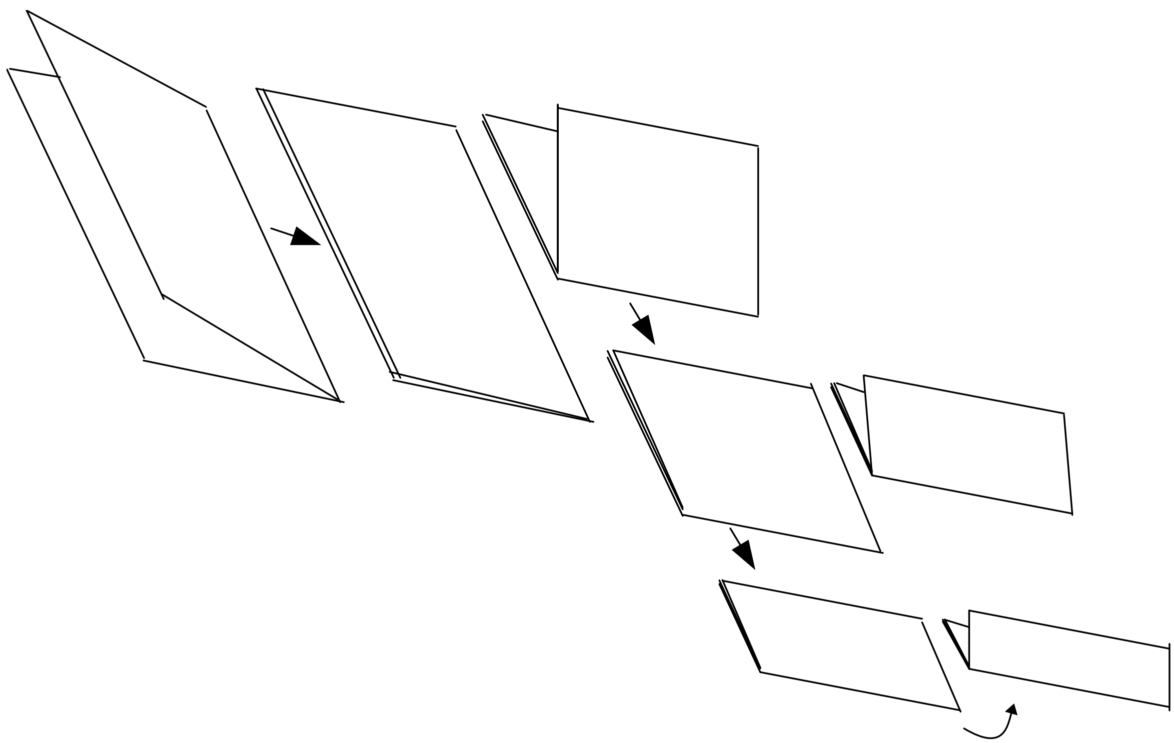
Lesson #13: Stepping Through Paper

Overview: Did you know that you can step through a sheet of paper using only a pair of scissors to help? Does this sound impossible? Great – let's get started.

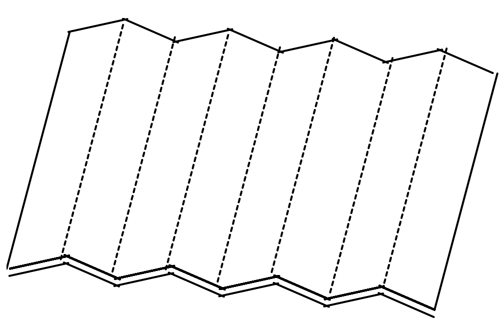
Materials

- A sheet of paper
- A pair of scissors

Activity: The first thing you need to do is fold your sheet of paper into two equal parts in a "hot dog" fold so it's 11" x 4.25". Now fold it again in half to have a quarter of the sheet of paper. Continue to fold to the smallest size as possible as shown in the figure below. (You'll probably want to watch the video for this part.)

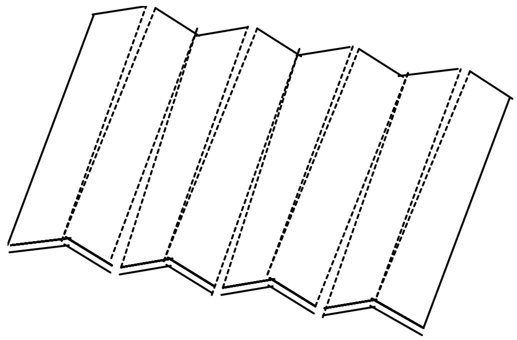


Unfold the paper to get this shape:

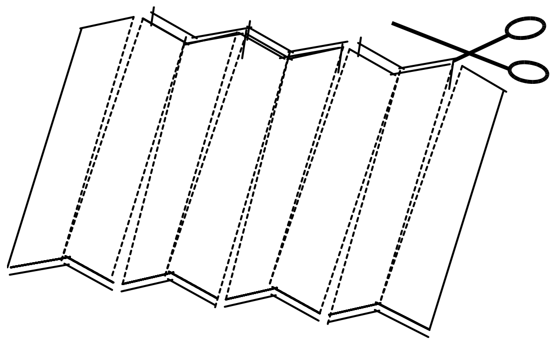


Next, use the pair of scissors to cut the folded side of the paper along the folds. IMPORTANT! Cut every other fold as shown in the video. Be careful not to cut all the way through! Leave the ends uncut as shown below:

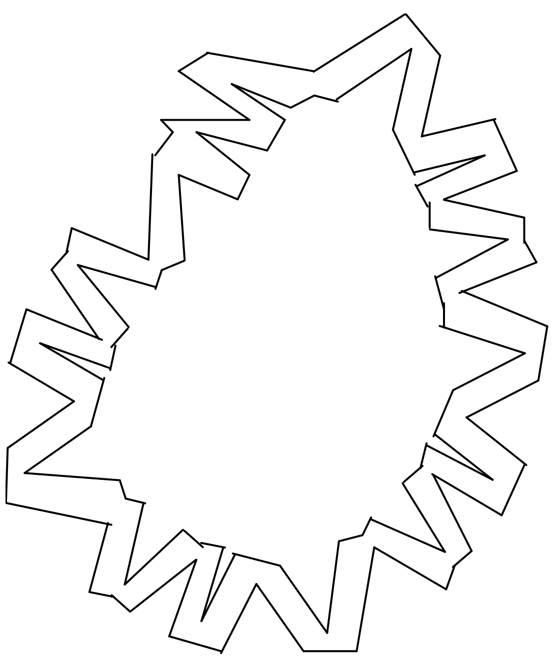
Next we cut the skipped folds from the opposite side as shown in the figure below. Again, don't cut all the way through!



Now cut the folded sides as shown below. Do not cut the first and the last folds!

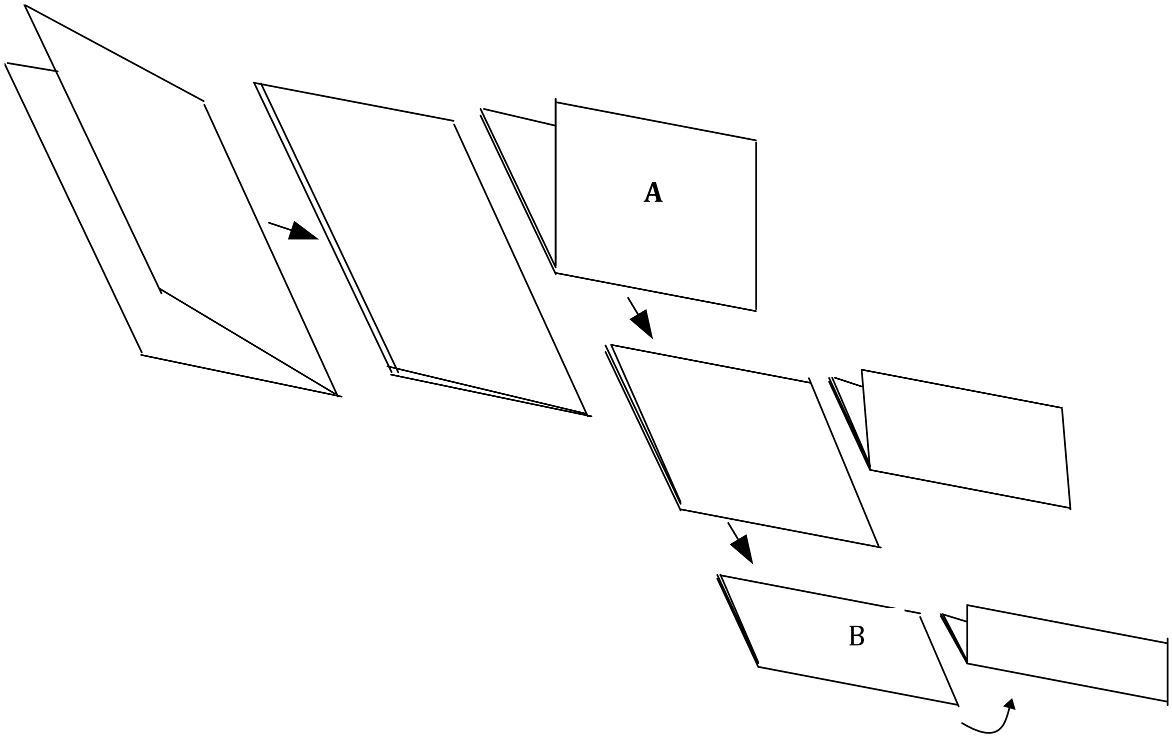


After the process, we come up with a connected strip of paper as shown below. Depending of the thickness of the folding, we can come up with a long strip that's big enough to step through!

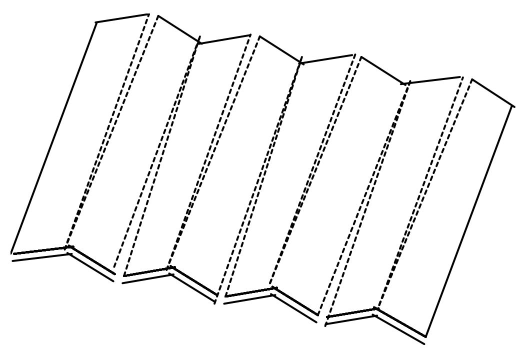


Exercises

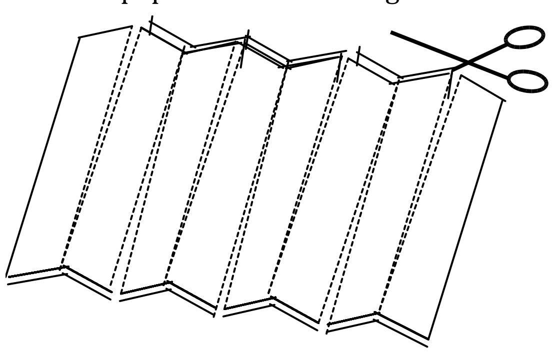
Use the diagram below to answer questions 1 to 6



1. How many folds are made according to the following diagrams?
2. What fraction of the paper is visible after the first fold?
3. What fraction of the paper is visible after the fourth fold?
4. What fraction of the paper is visible after the second fold?
5. How many fold lines does A have?
6. How many fold lines does B have?
7. In the second to last process, why don't we want to cut the first and the last folds?
8. If the paper shown in the figure to the right was to be cut end to end along the folds, how many smaller pieces of paper would result?



9. If the paper shown in the figure below was to be cut end to end along the folds then cut by the scissors as shown below, how many smaller pieces of paper would result?



Lesson #14: Sizes of geometry

Overview: Remembering and visualizing most shapes is pretty easy, right? An octagon can be a challenge for some (it has eight sides, while the commonly-confused hexagon has six sides). In this activity, we are going to use our memory to try to recall and draw some everyday objects such as a quarter, a playing card, and more, at their actual size. What objects around your house can you think of to use and test yourself?

Materials

- Pencil
- Paper
- A dollar bill, a button, a playing card, an eraser, or any other random item

Activity: For each of the following listed below, draw each one to scale to the best of your ability *without* looking at the original object first.

1. dollar bill
2. button
3. playing card
4. quarter
5. can opener
6. piece of chewing gum (before it's chewed)
7. paperclip

8. eraser

9. stick of butter

10. pinkie toe

So, how did you do? Were you able to get the size of your dollar pretty close to its actual size? How many objects were you able to closely estimate the size of in your sketches?

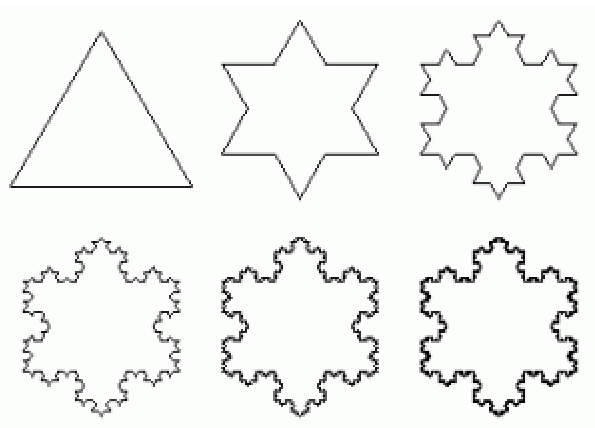
Lesson #15: Fractals

Overview: *Fractals* are new on the mathematics scene, however they are in your life every day. Cell phones use fractal antennas, doctors study fractal-based blood flow diagrams to search for cancerous cells, biologists use fractal theory to determine how much carbon dioxide an entire rain forest can absorb.

Fractals are in the mountains, clouds, coastlines, central nervous system, flower petals, sea shells, spider webs… they're everywhere! And the really nifty thing about fractals is that they are not only cool, they're super-useful in our world today.

Many mathematicians today are building on the work pioneered by Karl Weierstrass (1872), Helge von Koch (1904), and Waclaw Sierpinski (1915) to figure ways of using the ideas behind fractals. One of the most interesting parts about fractals is that many ideas about fractals were first thought up of in our lifetime. Many different fields, including medicine, business, geology, clothing fashion, art, and music use ideas about fractals.

Fractals are beautiful (there is something hauntingly stunning about the computer-generated images of objects such as the Mandelbrot set, Julia sets, the Koch snowflake). But that's not all – they are useful in our technology world. However, you'll find that many research mathematicians still roll their eyes at the mention of the word "fractal," mostly because the discussions you'll find out there concerning fractals are missing the most important element – the mathematical content! This is why you'll often find both students and teachers thinking that fractals are reserved only for art and video games, when that's only one side of a multi-faceted concept.

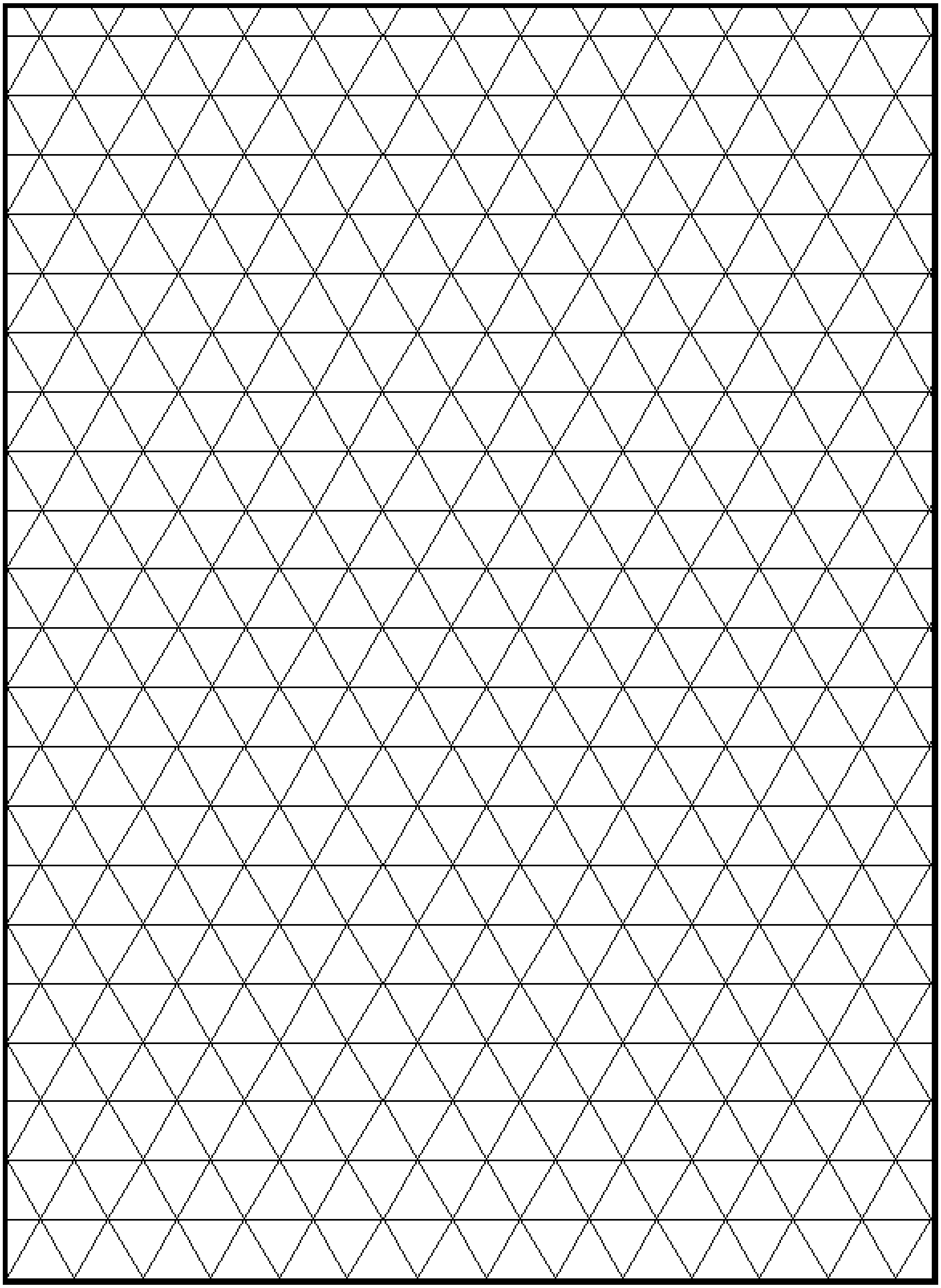


There is solid mathematics behind the pretty pictures – in fact, with a good program, most kids can create their own fractal images after starting with the mathematics (which is often more beautiful than the images themselves!)

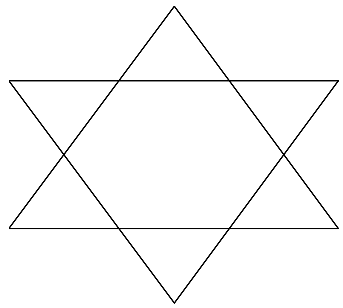
I'm going to help you unravel some of the mystery of fractals while having a lot of fun doing it. There are lots of easy-to-teach topics involving ideas from fractal geometry.

Materials

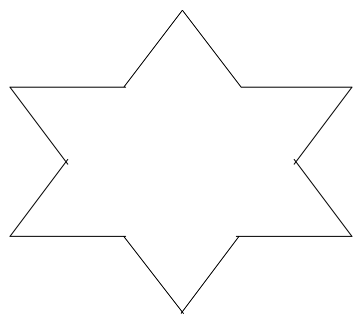
- Pencil
- Paper
- Grid made of triangles



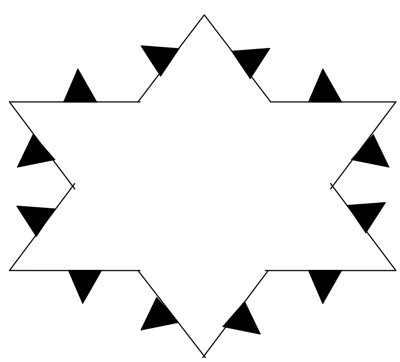
Activity: Look at how I can put together two triangles to make a six-sided star:



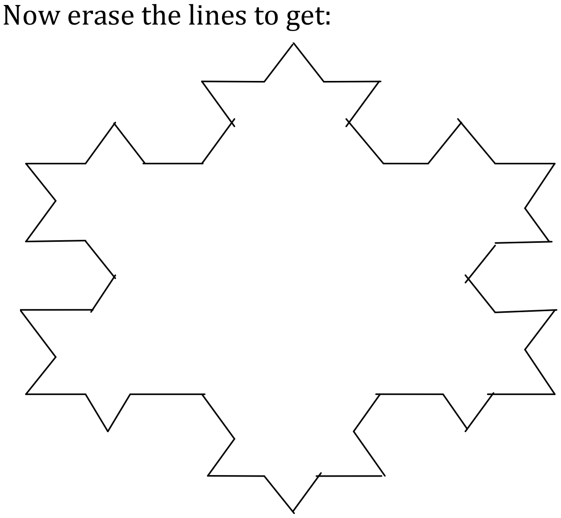
If I erase the inner lines of the triangle, I get the figure shown below:



Now find the middle of each line and add another triangle to each midpoint (dark triangles).



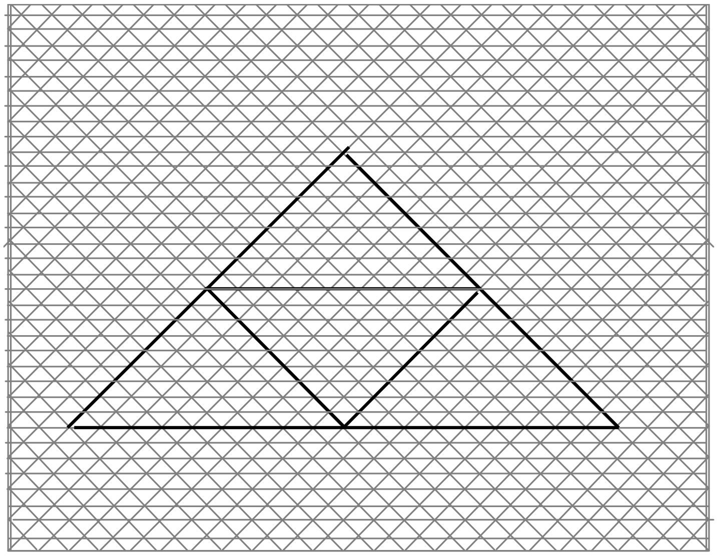
Now erase the lines to get:



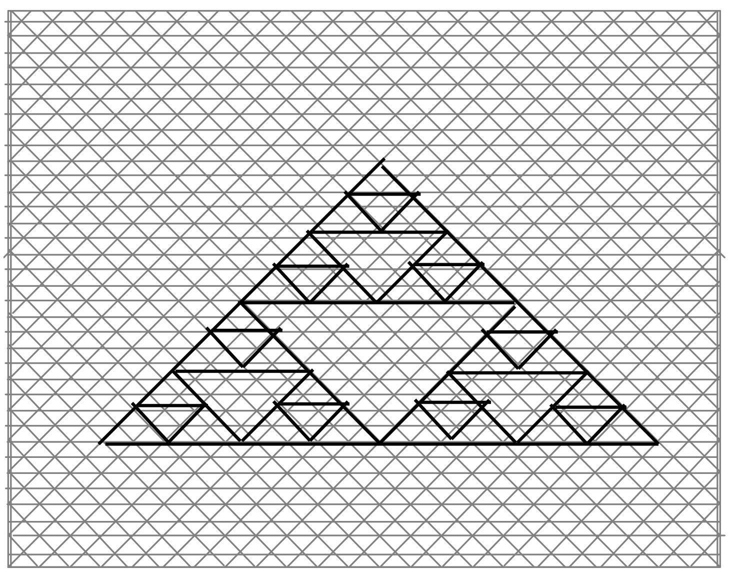
When we continue with adding more and more triangles at the midpoints of every line, you'll discover a complex and detailed structure similar to the one shown here. These are fractals!

The more triangles you add, the more the lines assume a more curved line.

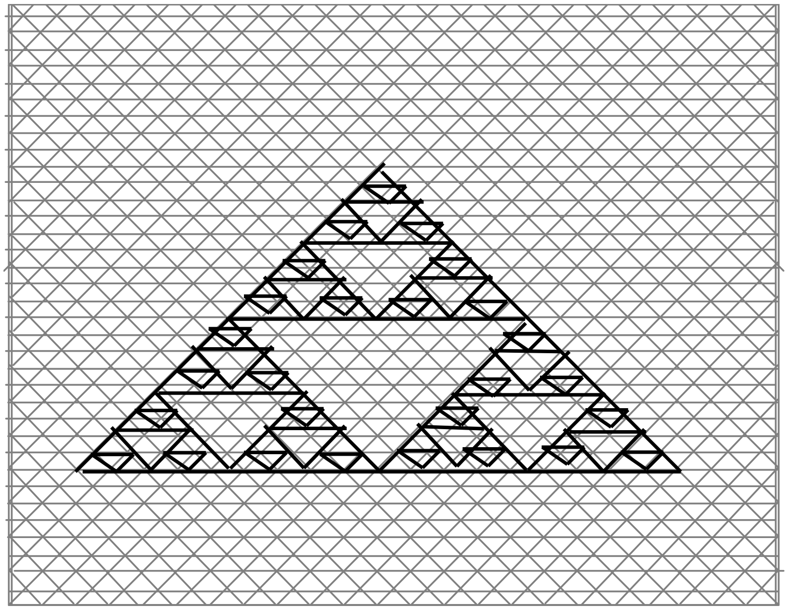
Let's try another way to create a fractal. Use the triangle paper to draw an equilateral triangle. Then divide it into three sections as shown:



Ignore the middle triangle, and now divide the other three smaller triangles as shown:



Again, ignore the middle triangles then divide the other smaller triangles as shown:

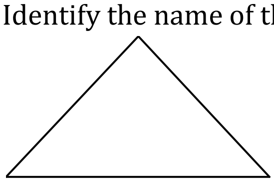


Continue doing this until you make the triangles as small as you can. You've just made a fractal! Notice that in every step, we ignored the middle triangle(s) and then divide the remaining triangles. There's a mathematical formula that can figure out the total area of the un-shaded (ignored) sections of a triangle.

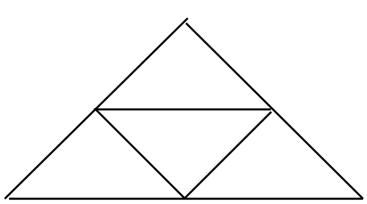
This particular triangle is called *Sierpinsiki* triangle, and it's widely used in structural engineering to model strong structural objects.

Exercises

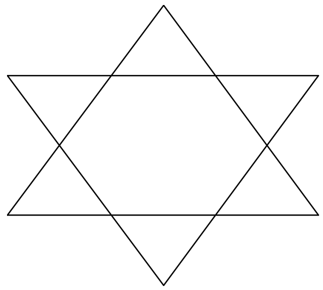
1. What are the best structures that can describe the structure of human hand, flowers, and peaks of mountains among others?
2. What are the building units of fractals?
3. Identify the name of the following figure.



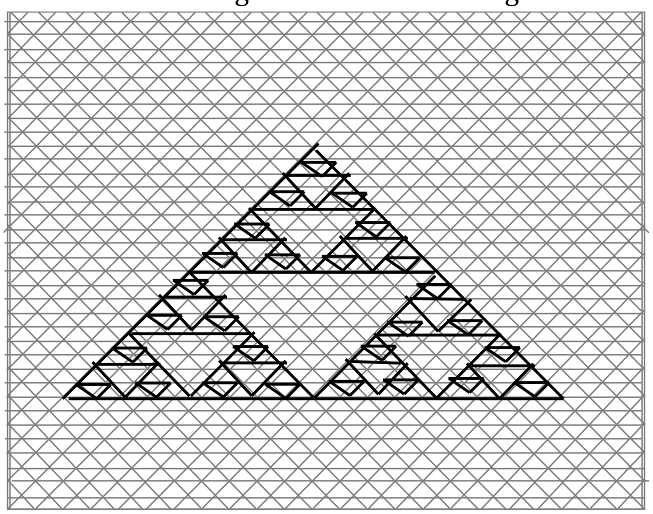
4. From the above lessons, what is the best type of three-sided figure that can model fractals. How many triangles are you able to identify in the following figures?
5.



6.



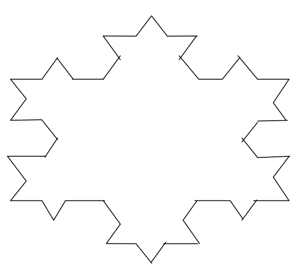
7. What is the name given to the following structure



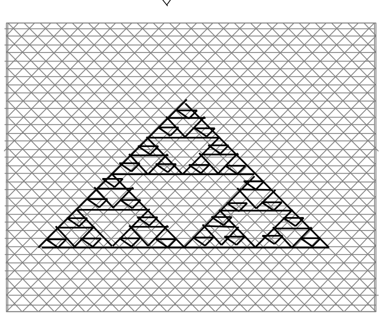
8. Where is the structure commonly used?

What are the basic procedures that are used in making the following fractals in the following two examples?

9.



10.



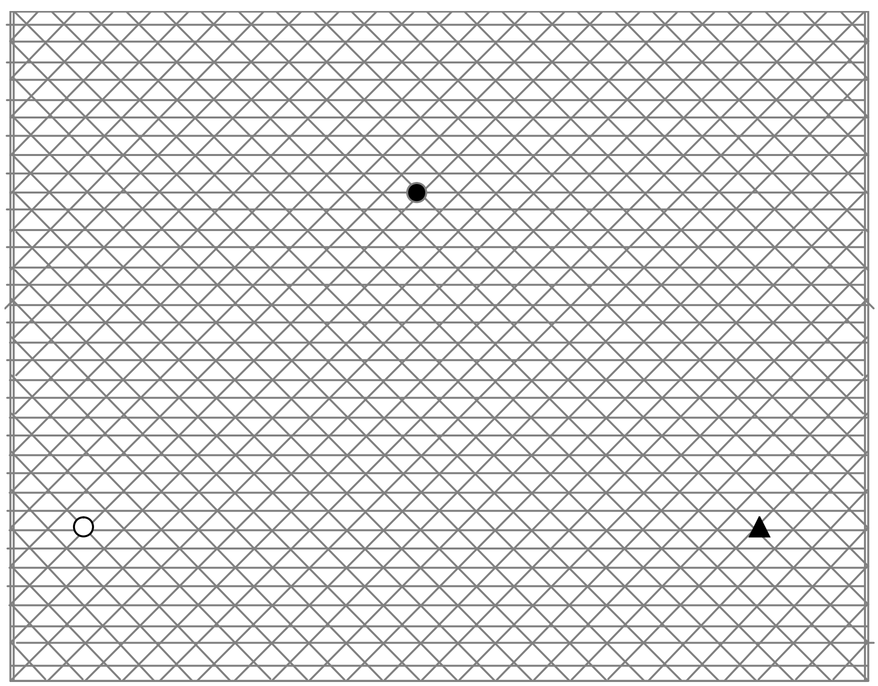
Lesson #16: Chaos Fractal game

Overview: This is a game that is based on fractals. You'll need the triangle grid paper and a die. This is a great demonstration on how probability can be used can be used with a game to come up with beautiful patterns.

Materials

- Three pens with different colors, possible blue, green and red
- Dice (well, really just one die that you are okay with drawing on to color-code it)
- A paper with triangular grids (refer to last pages of this lesson)

Activity: On the grid paper, mark three dots with a different color each: red, blue and green. In the sample below (since we don't have colors, just black and white) you'll find a solid triangle, a solid circle and an unshaded circle to mark the three different "colors." The dots should be the vertices of an equilateral triangle.



We need to convert the six-sided die into a die that has only three options (instead of six). Use your markers to color two sides of the die the same color. Do this for all three colors, so you've shaded two faces blue, two faces red, and two faces green.

For our demonstration, blue is the solid triangle, red is the open circle and green is the solid circle.

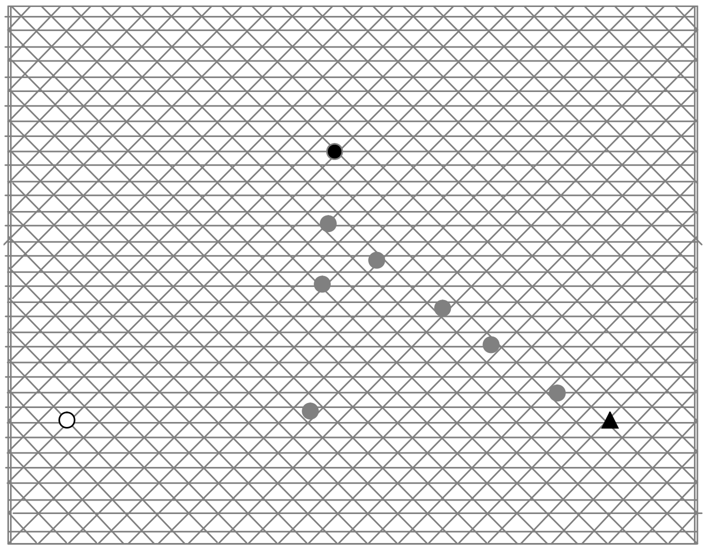
Use another color and mark a "seed point" – any point within the area bounded by the triangle. It can be anywhere – it really doesn't matter where. Just mark a spot.

Now roll the die. Suppose it lands on a blue face. Now measure out the distance (it doesn't have to be perfect – you can eyeball it) between the seed point and the blue dot and move your seed point to this new location by making a new dot (leave the original seed mark there).

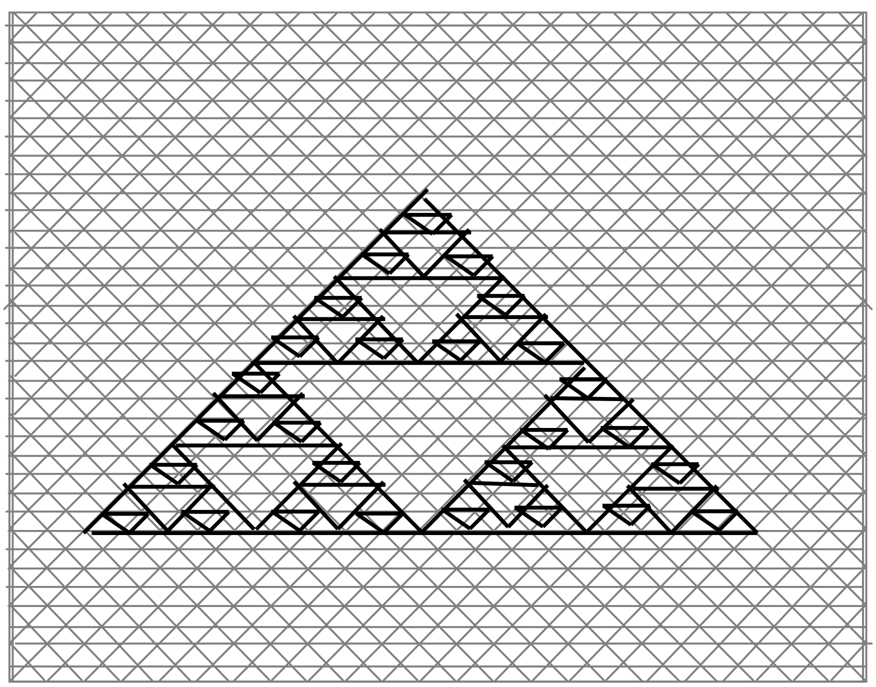
Now roll the die again. Suppose you get blue again. Now mark another spot that's halfway between your most recent dot and the blue vertex of the original triangle.

If you roll again and you get red, then you look at the distance between the red vertex (the open circle in our diagram) and the most recent mark.

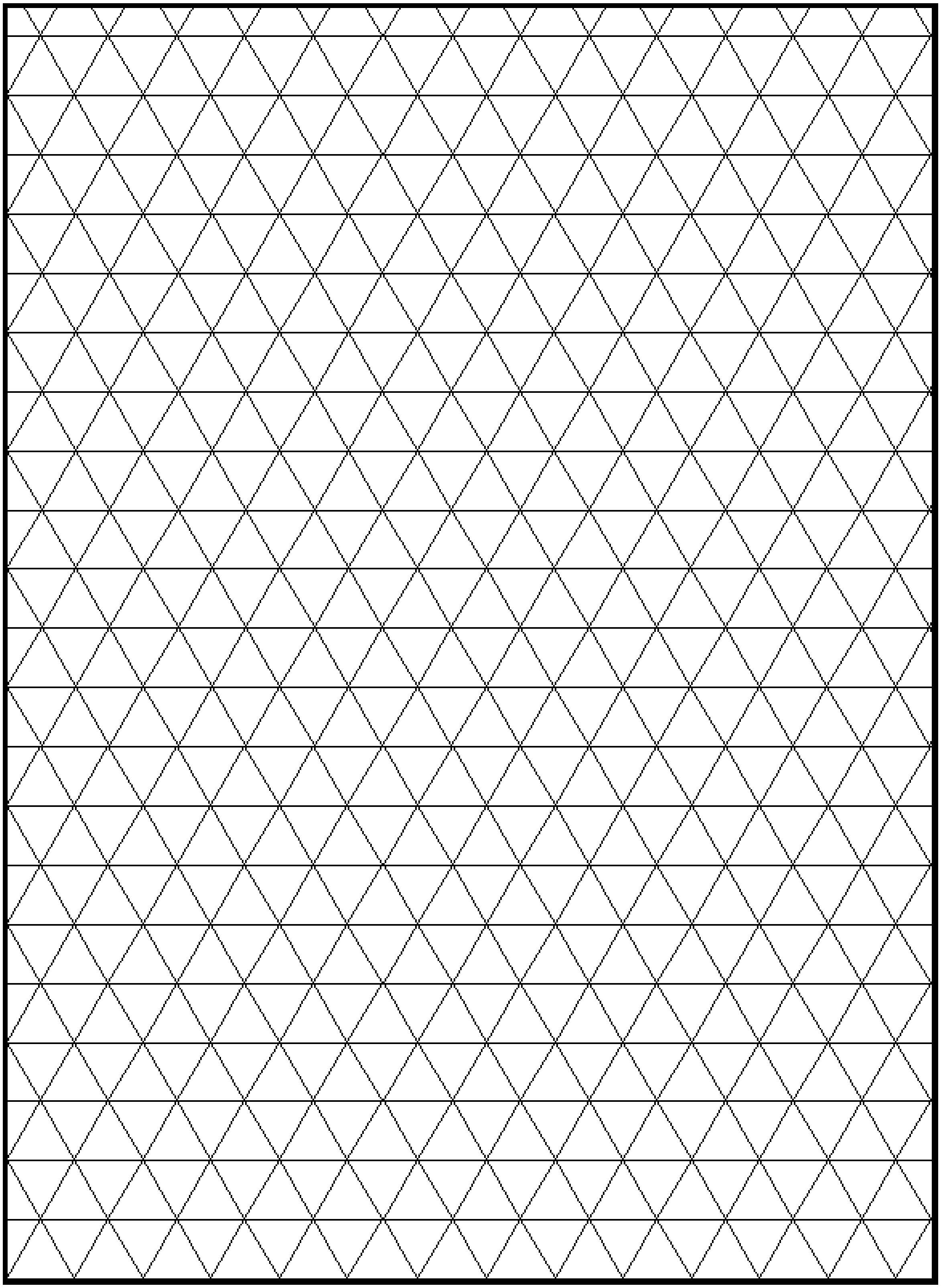
After several rolls of the die, you might get something like this:



Imagine that you roll the die 100 times, and make 100 different dots. What do you think the shape would be that the dots would make? (Hint: Think *Sierpinsiki* triangle!)



The chances of rolling a red are one in three, or one third. This game uses both probability and fractals in one neat application. You can color code your dots such that when you roll a red, use your red marker to make the new point on your graph. If you roll a green, mark the dot with a green marker, and so on.



Exercises

1. How many players can comfortably play the fractals game at a time?
2. How many ink pen colors are required to effectively play the game?
3. When are the first dots made on the grid?
4. What are the building units of the fractal grid that is used in the game?
5. Where specifically are the first dots placed?
6. Apart from the pens and the grid, what else is required to play the game?
7. What is the probability that at least one player will play in a given turn?
8. What is done on the die to ensure equal chances for each player?
9. In the subsequent plays, where is the dot corresponding to the player positioned?
10. What is the name of the resultant figure that results if the player goes through 100 turns?

Lesson #17: Real Geometry: The Pantograph

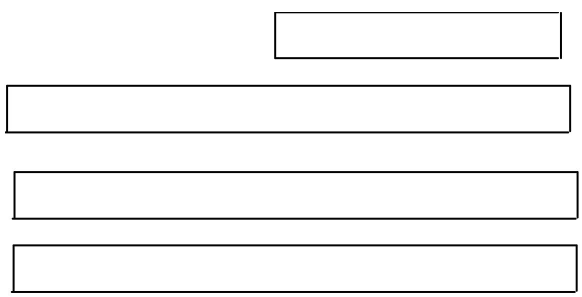
Overview: A pantograph, first invented in the early 1600s, was used to make exact copies before there were any Xerox machines around. It's a simple mechanical device made up of four bars linked together in a parallelogram shape.

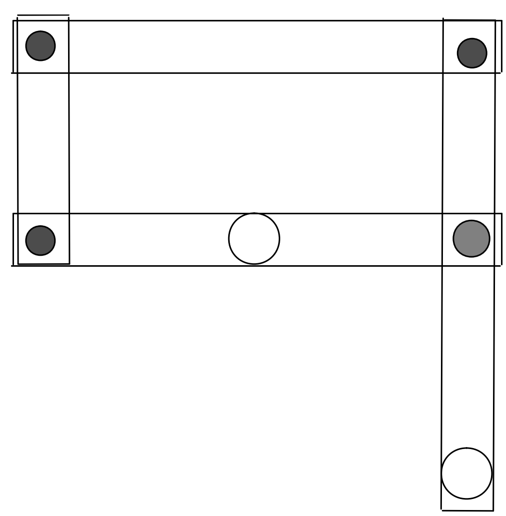
Here's how it works: By simply tracing an object with the pointer, the pantograph makes a copy larger or smaller depending on which point you attach your pen and pointer. Some pantographs were adjustable – meaning that they could change their pivot points to adjust the size of the copies. We're going to make one of these to see how geometry can really be used in the real world.

Materials

- Paper
- 2 mechanical pencils
- Masking tape
- 4 brass fasteners
- 2 yardsticks
- Strong scissors or saw to cut the yardsticks into three 16″ lengths and one 8″ length
- Drill with drill bits
- Scrap piece of cardboard, wood, or other old table space to practice on (your table may get scratched)

Activity: To assemble the pantograph, first divide the yardsticks into three 16-inch pieces and one 8-inch piece.





Drill holes into the sticks and assemble them so that they form a rectangle as shown in the figure.

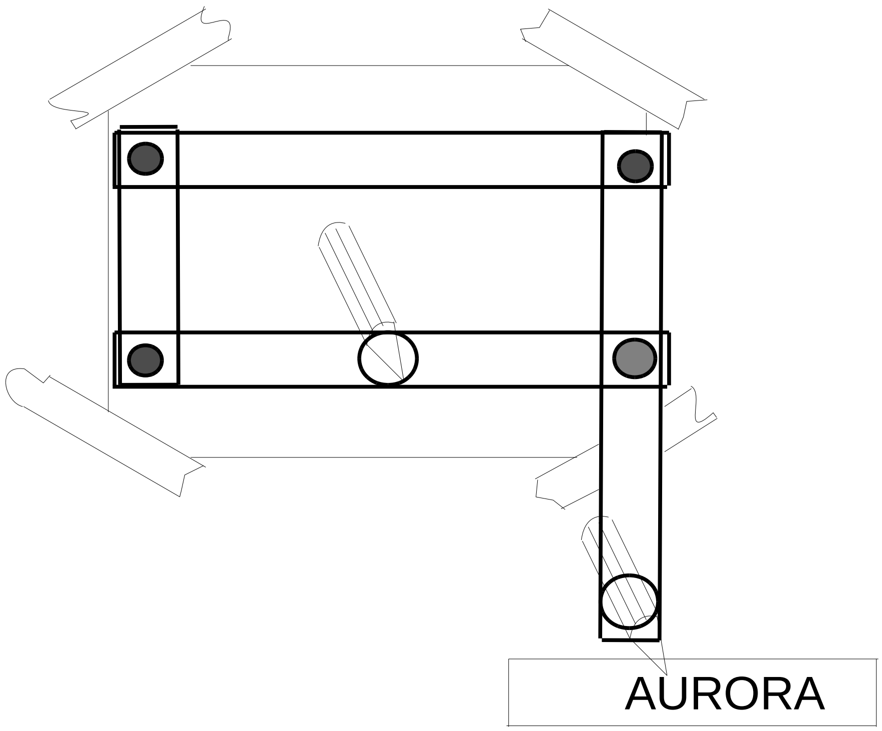
Insert the brass fasteners into the four shaded holes. If the holes aren't large enough so that the brass fasteners can fit freely without falling through, then drill the holes out a little more.

Drill two more holes (shown by the open circles in the image) *slightly* smaller than the diameter of the pencils. Insert pencils into each of the holes – you should need to press-fit them in. If your holes are too large, wrap your pencils with tape to fill in the gap.

The pencil in the horizontal bar is for drawing, and the pencil in the vertical bar is for pointing (called the *pointing pencil* or *pointer*).

Mount a blank sheet of paper on a drawing board using the tape.

On a second sheet of paper, write your name. I wrote AURORA on mine. Put the name under the pointer.

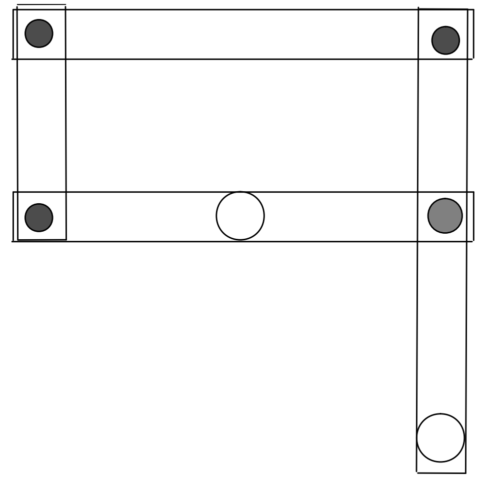


To use the pantograph, hold the top left joint in a fixed position and use the pointer to trace the name. Make sure your drawing pencil is touching the paper enough to leave a mark so you can see what it's writing!

Which one (the drawing pencil or pointer) makes the smaller drawing?

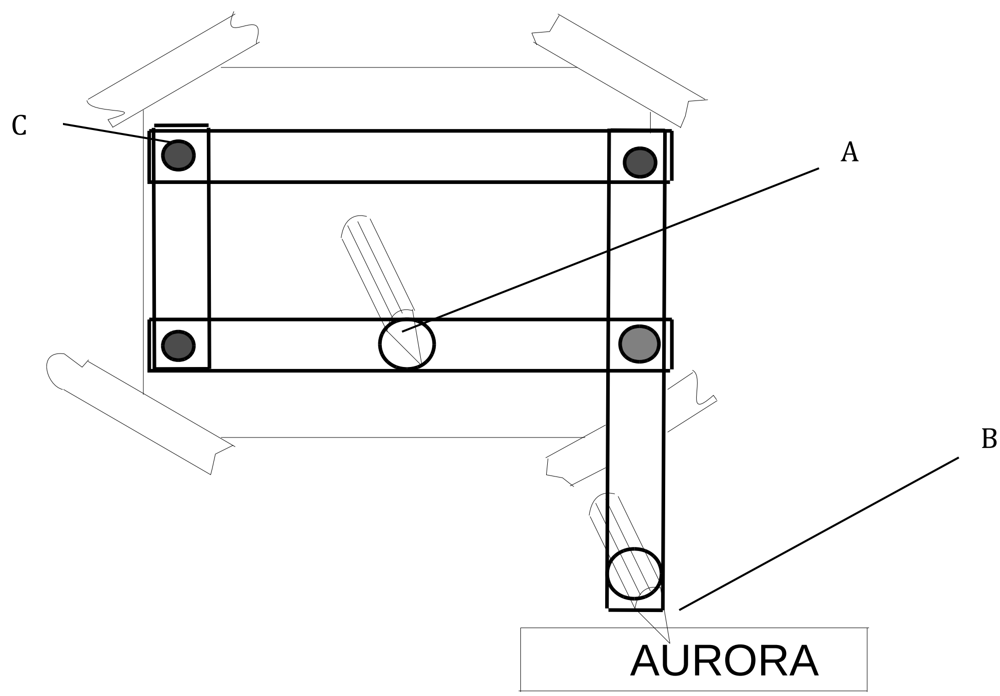
Exercises

1. Identify the name of the image at the right:



2. What is the use of the instrument shown?
3. Why is the drill necessary in assembling the instrument?
4. What is the purpose of the masking tape in the above procedure?
5. Identify the names of the two pencils as used in the drawing procedure.

Use the diagram below to answer question 6, 7 and 8.



6. What is the size of the object at B as compared to that of the original object?
7. What is the size of the object at A as compared to that of the original object?
8. How is the joint at C treated to allow the instrument to work as designed?
9. How are the holes at the joints treated so ensure the instrument works efficiently?
10. When the instrument is at work, what kind of geometrical figure does is formed between the horizontal and the vertical bars?

Lesson #18: Graphical multiplication

Overview: The trick looks impressive, so be prepared for jaw-drops when you show this to kids and adults. But can you figure out how it works? I'll give you a hint: Think about how to represent placeholders of powers of 10...

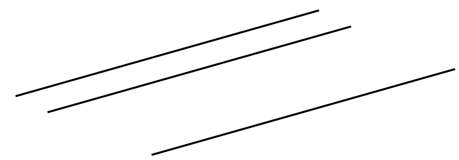
Materials

- Pencil
- Paper

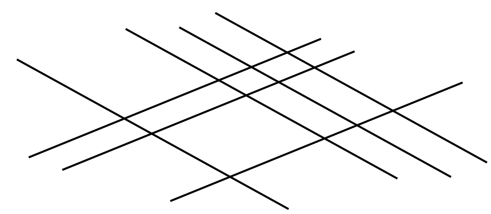
Activity: This is a great method for kids who might have difficulty understanding multiplication, since it's graphical. Give it a try!

Let's multiply two numbers together: 21 x 13 =?

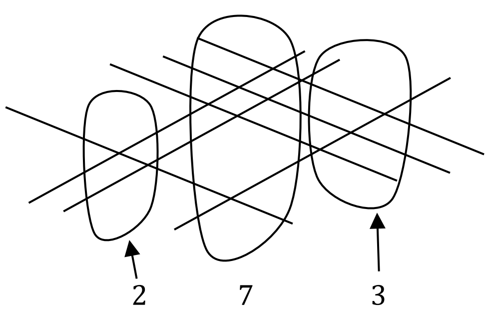
For 21, we draw two lines for the tens place and one line for the ones place:



Then draw the lines that represent 13. First, draw one line for the tens digit and three lines for the ones digit like this:



Now we count up the intersections. First, group them together vertically like this, and then count the number of intersections in each group.

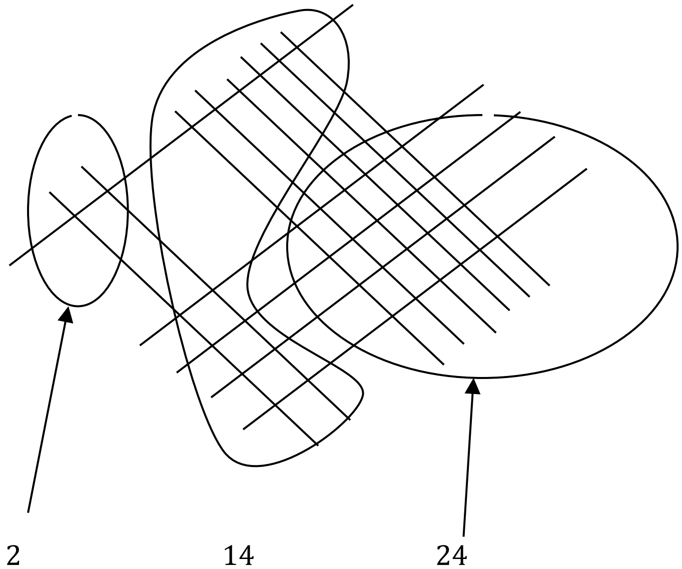


Put together the numbers that you counted up from the intersections (the 2, 7, and 3) to get the answer to the problem… 273.

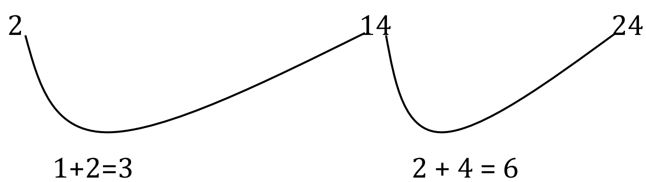
So 21 x 13 = 273!

Now you try this one before turning the page: 14 x 26 =?

First draw the lines for 14 and 26 as shown below, and then count the intersections.



The answer to 14 x 16 is not 21,424, since that's a five-digit answer, and we expect ours to be only three digits (as we've discussed before). So, since some of the numbers are more than 10, we have to perform a "carry" forward like this:

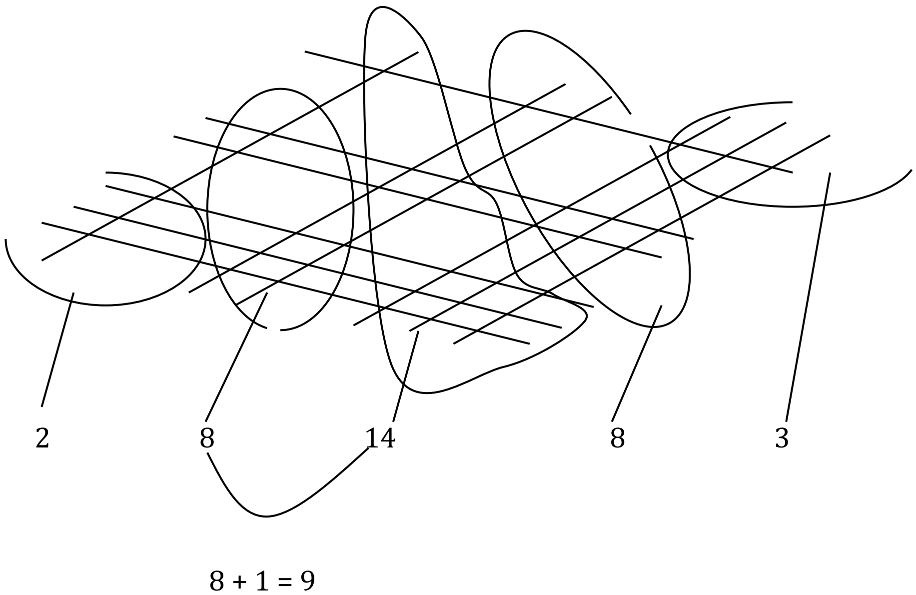


The answer becomes: 364.

So 14 x 26 = <u>364</u>!

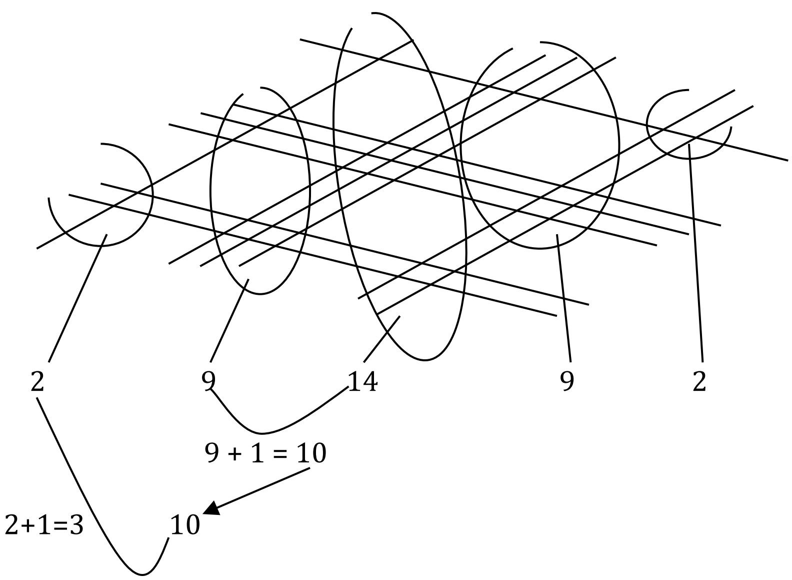
What about multiplying three-digit numbers? Well, you really just need a larger sheet of paper.

For example, let's figure out : 123 x 321 = ?



The solution is 123 x 321 =29,483!

Let's figure out: 132 x 231 = ?



The solution is: 132 x 231 = 30,492!

How does this work? For the first example, we represented the number 21 with a set of two lines and then with one line. Then we turned 90 degrees and added the number 13 on top by drawing one line followed by a set of three lines.

But what do the lines really mean? Remember my hint about placeholders? Well, the lines are really placeholders for the following multiplication:

$21 \times 13 = (2\times10 + 1)(1\times10 + 3)$

Instead of writing out the numbers like the problem above, we simply draw lines to mean the same thing. If you were to cross-multiply that problem, we get a scary thing that looks like this when we group it in powers of ten:

$(2\times10 + 1)(1\times10 + 3)= 2*10^2 + (2\times3 +1) \times 10 + 3 = 273$

The answer of 273 comes from figuring out there are 2 units of 100 (or 10^2), 7 units of 10, and 3 units of 1. Those are the intersection points of the lines we drew.

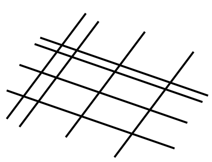
Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

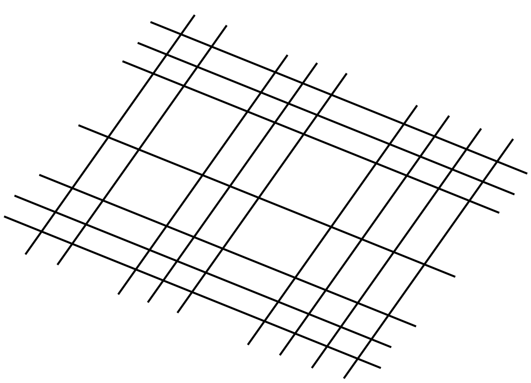
1. 23 x 45
2. 56 x 72
3. 52 x 26
4. 62 x 49
5. 67 x 92

For the following, write out the problems to solve and then solve them:

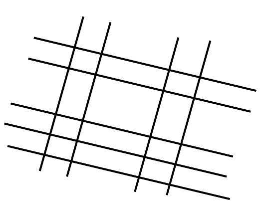
6.



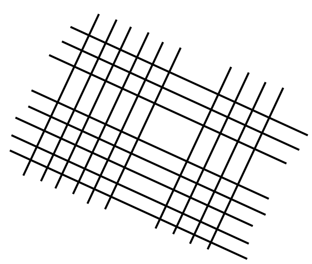
7.



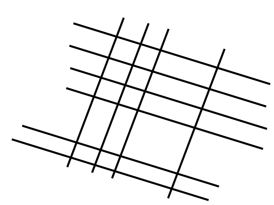
8.



9.



10.



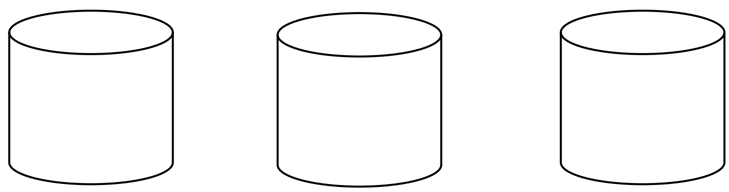
Lesson #19: Paperclip trick

Overview: Math isn't just about numbers or shapes… it's also about games and thinking about things in a new way. This problem is a great example of this. It's a neat logic question that also involves some spatial thinking, like geometry, to work yourself out of what seems like a paradox. Before you look at the solution on the next page, see if you can figure it out for yourself.

Materials

- 11 paper clips
- Three cups

Activity: For this puzzle, you'll use three cups and 11 objects. The first challenge is to put an odd number of objects in each cup. Is this pretty simple? How many different combinations can you come up with for the 11 objects?

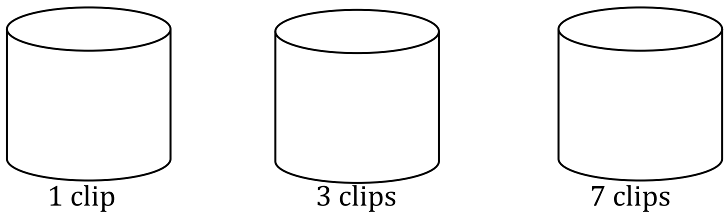


Now take one paperclip away and use ten instead. Is this puzzle more challenging?

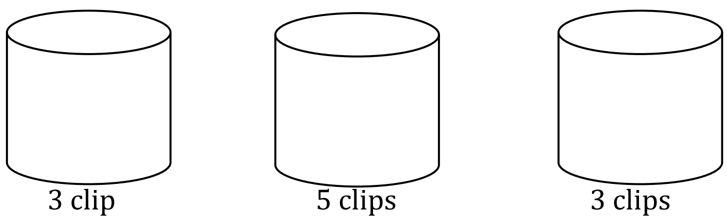
Try to figure this out for yourself first before turning the page! Make notes here on things you tried that did and didn't work:

Need a hint? Can you figure out a way for one paperclip to be in two different cups at the same time?

Solution time! For 11 paperclips, you'll find this is one solution to the problem:



Another solution is here:



Pretty cool, right? I hope you enjoyed this trick. Mathematics and paradoxes can really be a lot of fun!

Exercises

1. What is an even number?
2. What is an odd number?

If you have 8 paperclips and three cups, arrange the paperclips you that you have an odd number in each cup. Find three different solutions (arrangements) to solve this problem.

3. First arrangement
4. Second arrangement
5. Third arrangement

Now you have 11 paperclips and three cups. Find two *new* solutions (ones we haven't covered yet in this lesson) to having an odd number in each cup.

6. First arrangement
7. Second arrangement

Reduce your number to 10 paperclips. Find four *new* solutions (ones we haven't covered yet in this lesson) to having an odd number in each cup. (Is it easier this time?)

8. First arrangement
9. Second arrangement
10. Third arrangement
11. Fourth arrangement

Lesson #20: New Year's Puzzle

Overview: This is a really fun calendar riddle I learned back when I was a student that really had me going for days before I figured it out. This one really seems like a paradox at first. It's a math logic puzzle that will really blow your mind. Are you ready?

Materials

- Pencil
- Paper

Activity: On New Year's Day (Jan. 1st), some friends came over to visit and they asked you, "How old are you?"

To which you replied, "The day before yesterday, I was nine years old. And next year I am going to be 12."

And this is totally true.

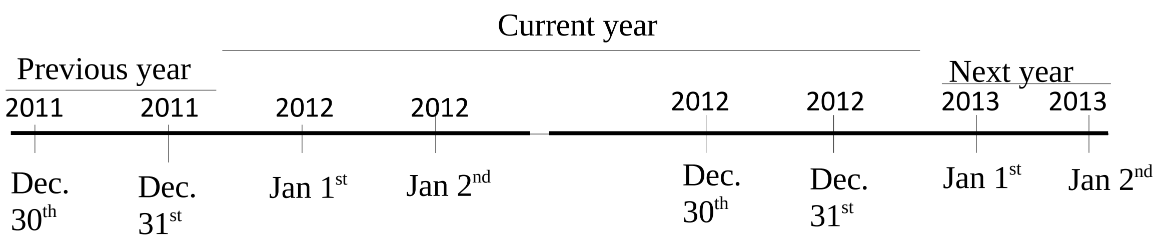
How can this be?

I'll give you a hint… your birthday is on Dec. 31st. Can you explain the rest?

Use the space below to figure out what's going on. When you're ready for the solution, turn the page. *(No peeking until you've tried it first!)*

Need a hint? Try sketching out a timeline right around Dec. 31 and Jan. 1, keeping track of the year.

Did you solve it yet? Let me give you a little help if you need it. Assume today is Jan. 1st (New Year's Day) in the year 2012. Your birthday on Dec. 31st. Now check out the diagram:



Remember you said, "The day before yesterday, I was nine years old." Which day are you referring to?

The day before yesterday is Dec. 30th, 2011. That's the day you were 9.

The following day (yesterday, Dec. 31st, 2011) you turned 10. With me so far? Good.

How old will you be on Dec. 31st, 2012? Eleven years old. But that's not what you were referring to, is it? Because you also said, "And next year I am going to be 12." You said that *on* Jan. 1, 2012, so you couldn't have been talking about your birthday in 2012, but rather your birthday in 2013!

How old will you be on Dec. 31st, 2013?

You'll be 12 years old. *Ta-daa!*

Exercises

Your exercise for this lesson is to not only challenge someone else with this problem, but be able to explain it to them in a way that they understand the solution. Go for it!

Lesson #21: Logic Numbers

Overview: This is a neat logic trick which allows you to flip over a stack of cards numbered 1-10. When you flip them back upright, they are in numerical order. There is a special way to make it work, so pay close attention!

Materials

- Pencil
- Paper or 10 index cards

Activity: First, make yourself a set of 10 cards, each with a number on it. Label the first card with a “1,” the second with a “2,” and so forth on up to “10.” If you’re using paper, you’ll need 10 sheets, or cut the paper into 10 equal-sized pieces before you start.

Now we’re going to do a magic trick with the cards. It’s best to stop here and watch the video so you can see the trick in action.

Did you notice how I flipped the top card over and it was a 1? And then I didn’t reveal the second card, but instead put it at the bottom of the pile. The third card was flipped over and it was a 2. The fourth card went straight to the bottom of the deck, but the fifth card was a 3… and I continued to flip one over and place one on the bottom.

Did you also notice that all the cards came off in exactly the right sequential order? How did I do that? Pure logical thinking!

My question is: What order do you think these cards need to go in order to do that?

Use the space below to figure out what order the cards need to be in for this puzzle:

(You’ll find the solution at the back of this book.)

Exercises

Your exercise for this lesson is to not only challenge someone else with this problem, but be able to explain it to them in a way that they understand the solution.

Lesson #22: Checkerboard paradox

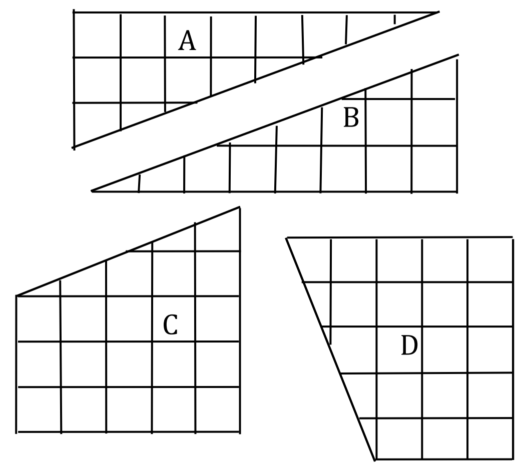
Overview: Once in a while, mathematicians come up against something that really seems impossible on the surface. These seeming "impossibilities" not only cause them to sit up and take notice, but often to create new rules about the way math works, or at the very least understand math a little better.

Be warned, however, that some paradoxes are really false paradoxes, because they do not present ***actual*** contradictions, and are merely "slick logic" tricks. Other paradoxes are real, and these are the ones that shake the entire world of mathematics. There are several paradoxes that remain unsolved today.

Materials

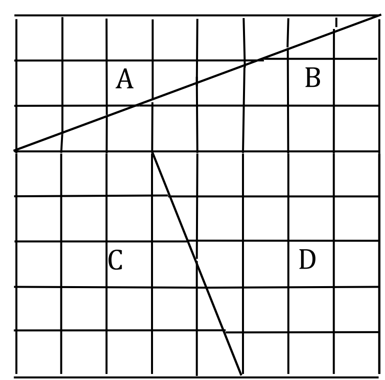
- Pencil
- Template from this page

Activity Cut out the four shapes (A, B, C, and D) so they look like this:

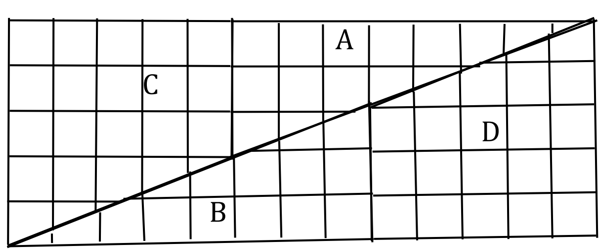


Now can you puzzle the pieces together to make a square? How many little squares are in the big one? ____________

Did you say 64? You're right! Since this is a square of 8 units per side, 8 x 8 = 64 square units.



Now can you make a rectangle like the one below? How many little squares are in the big rectangle?___________

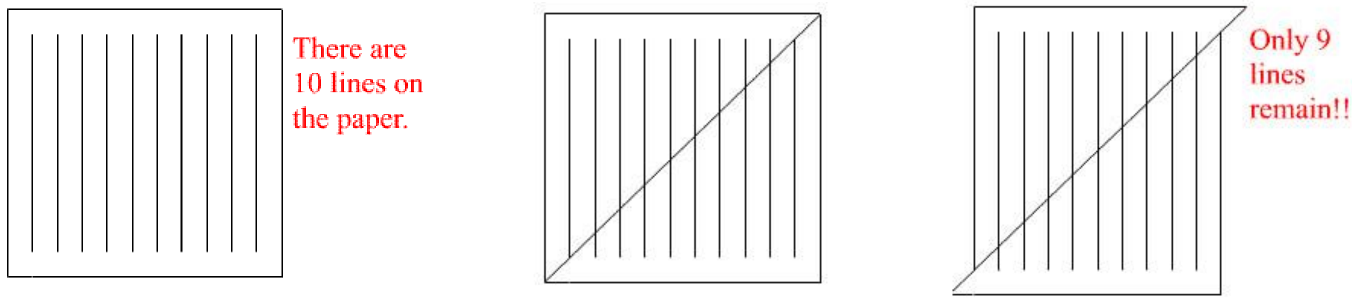


Uh-oh … did you get a different answer? What happened??

The paradox is that when you arrange the pieces in a square, you count up 64 units. When they are in a rectangle, a mysterious 65^{th} box appears.

(Try to figure this out yourself before turning the page!)

Need a hint? I'll give you a hint. Look at this picture:



If you cut the rectangle along the diagonal (the line that magically appears from one corner to the other) and then slide the lower triangle as shown, you can count the number of vertical lines and find that there are only nine! What happened to the tenth?

You can make it magically appear if you slide the lower triangle back to its original position. So… my question to you is: Which is the line that has returned and where does it come from?

The secret is this: there is a progressive decrease in the length of the segments above the diagonal and a corresponding increase in the length of segments below. What happens is that eight of the 10 lines are broken into two segments, then these 16 segments are redistributed to form nine lines, each a trifle longer than before. Because the increase in the length of each line is slight, it is not immediately noticeable. In fact, the total of all these small increases exactly equals the length of one of the original lines. Therefore, there is actually not a line which vanishes.

Now… how would you explain the checkerboard paradox?

Exercises

Your exercise for this lesson is to not only challenge someone else with this problem, but be able to explain it to them in a way that they understand the solution.

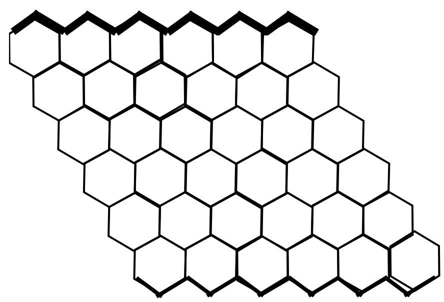
Lesson #23: Hex

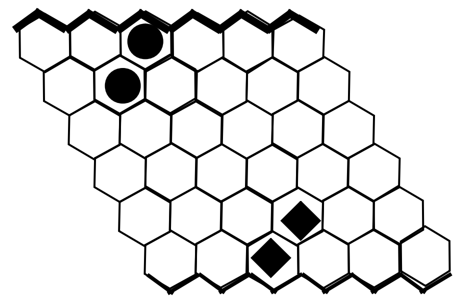
Overview: Hex is a super fun game! It starts with a grid of hexagons (six-sided shapes) and two players. You can color in any cell on your turn. The ultimate goal is to be the first one to complete a chain across to the other side of the board.

Using the pie rule can help with the advantage that the first player gets. This means the second player can choose to switch positions with the first player after they've made their first move. Can you use your logic skills to find strategies that make getting across the board easier?

Materials

- Two different colors of markers or crayons
- Hex board (refer to next page)



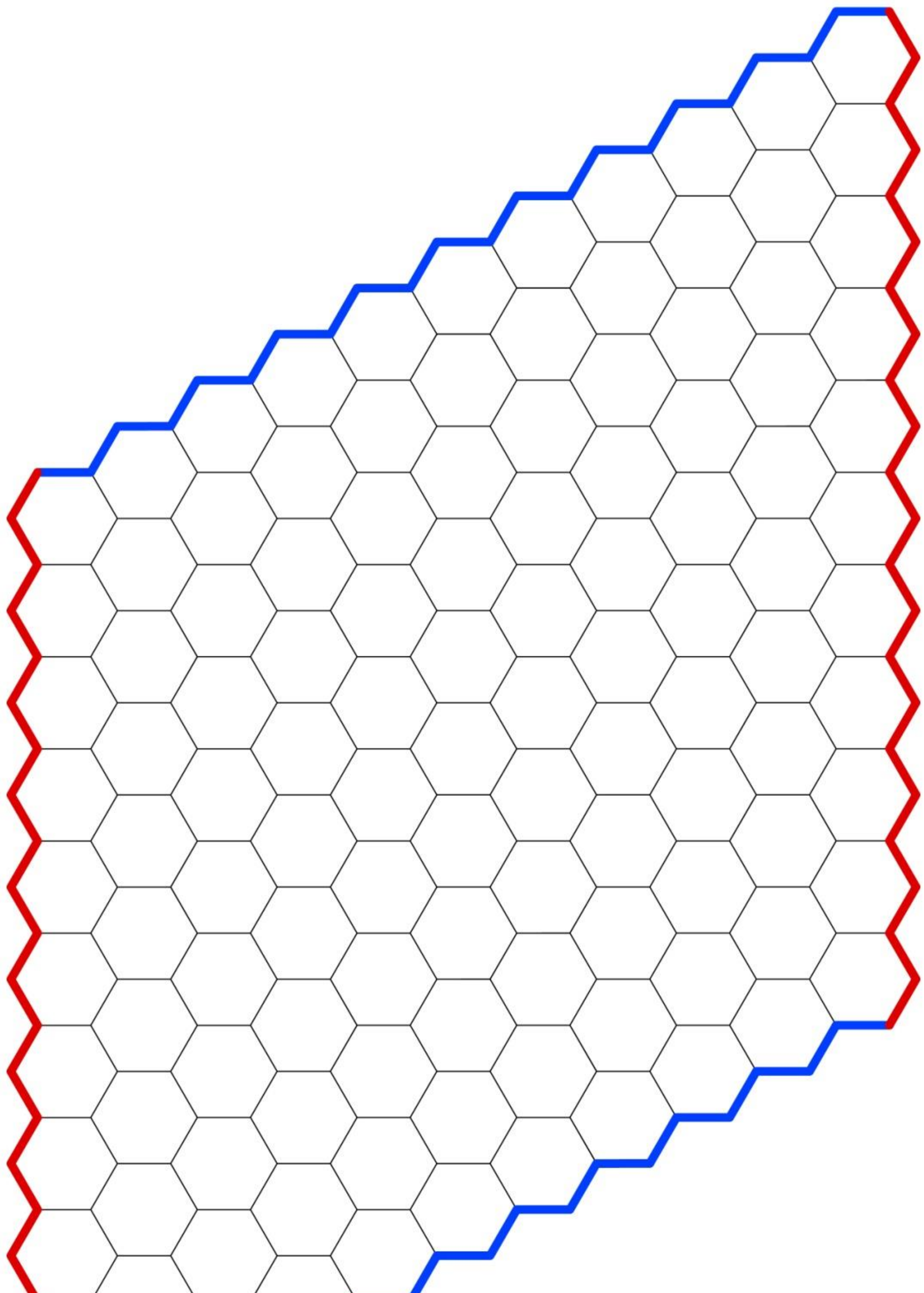


Activity: In the left image, the first player chooses to begin from the lower side of the board and draws with her color along the zigzag edge to mark her home base. The second player does the same at the opposite side with his color.

On the right image, the game has started.
Each player shades one of the hexagons on their turn, and it stays that color for the entire game. Either player can shade in an empty hexagon on their turn. *Any* hexagon on the entire board is up for grabs on your turn.

The goal is to connect a row of adjacent hexagons between your two sides. Any continuous path will work, and you don't have to shade in the hexagons in any particular order.

Give it a try!



Lesson #24: Magic Squares

Overview: Magic squares have been traced through history as known to Chinese mathematicians, Arab mathematicians, Indian and Egyptian cultures. Magic squares have fascinated people for centuries, and historians have found them engraved in stone or metal and worn as necklaces. Early cultures believed that wearing magic squares would ensure they had long life and kept them from getting sick.

Benjamin Franklin was well-known for creating and enjoying magic squares, and it was all the rage during his time. Here's the deal: We're going to arrange numbers in a way so that all the rows, columns, and even the diagonals add up to a single number (called a Magic Sum). The first Magic Square was published in Europe way back in 1514.

Materials

- Pencil
- Paper

Activity: Here's the first magic square, which was published in Europe in 1514. The square has the number 1514 embedded within it (can you find it?).

In the square, when you the sum the numbers in the boxes of any diagonal, row, or column, you always get the same answer: 34. For this particular square, you can add the corners and you'll also get 34.

34 is the "magic sum" of this magic square. Every magic square has its own magic sum. Some squares are 15, while others are much larger.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

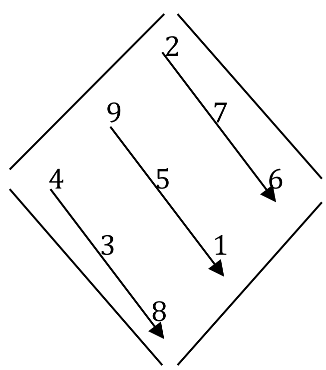
Let's build a magic square from scratch. Start by drawing a box of 9 squares, and write the numbers as shown:

1	2	3
4	5	6
7	8	9

Now swap the values at the corners as shown (the 9 and 1 trade places, as do the 3 and 7):

9	2	7
4	5	6
3	8	1

Now imagine that the grid is made out of rubber bands that can stretch and move. Pinch the 2 with one hand, the 8 with the other, and pull them apart to reform the grid into a diamond shape, with the 2 at the top and the 8 at the bottom. Do you see how I made the numbers in the image below doing this?



Now write the numbers along the arrows into a new grid like this:

2	7	6
9	5	1
4	3	8

This is your new magic square. What is the magic sum?

The sum of the elements on any diagonal, row or column is 15. Now erase 4 to 6 of the values (depending on the skill of your audience) in your box and give it to someone to solve. You've created your own magic square puzzle!

We can also have a look at the upside-down magic square, which gives a magic sum of 264:

96	11	89	68
88	69	91	16
61	86	18	99
19	96	66	81

It's called an upside-down square because when you rotate it a half turn, the result is still a magic square (with the right computer font). The amazing thing is that it still adds up to 264!

18	99	86	61
66	81	98	19
91	16	69	88
89	68	11	96

Exercise

Find the value of the letters; A, B,C and D in the following upside-down magic square

18	A		61
D	81	98	
91	B		88
C	68		96

1. A
2. B
3. C
4. D
5. What is the sum of elements of any diagonal in the first magical square published in 1514?
6. Find the element at the middle of a nine-element magic square.
7. Draw a nine-element magic square.

Find the values of b and c in the following magic square.

16		2	
c	10	11	8
	6	7	
		d	b

8. b
9. c
10. d

Lesson #25: Bagels

Overview: This is one of my family's favorites! It's a guessing game, but you can use logic and strategy in order to guess the numbers very quickly. I'll show you how to use the game to guess numbers even larger than three digits. Once you've mastered the strategies in this game, you'll never lose another game of *Mastermind* again.

Materials

- Two people with brains

Activity: I have a two-digit number in my mind. Can you figure out what it is?

YOU: 12

ME: Bagels! (This means the number and the position are wrong.)

YOU: 34

ME: Bagels!

YOU: 65

ME: Pico! (This means right number, wrong position.)

The trouble is, you don't know which number is the right one. So guess something that will tell you which number is right, like guessing 45.

YOU: 45

ME: Bagels!

Now you know that the 5 is not the right number, and you also know the position that the 6 is in – the ones place. You can guess a number like 26 to check.

YOU: 26

ME: Fermi! (This means that one number is the right one in the right position.)

YOU: 78

ME: Pico! (This means right number, wrong position.)

Since you already know that the ones place is takes by the 6, you also know the tens is the only one open. And since the response was "pico", you know it's the 8 since it wasn't in the right position.

YOU: 86!

ME: Double Fermi! (This means for a two-digit number game, you've won!)

What about more than two digits? Here's how you handle three and more digits:

YOU: 1111

ME: Fermi! (One of the 1's is in the right position.)

YOU: 1222

ME: Pico! (There is one right number in the wrong position, and there are no 2's in this number.)

YOU: 3133

ME: Double fermi! (So now the 1 and one of the 3's is in the right position, but we don't know which 3 yet.)

YOU: 4134

ME: Femi pico! (Now we know there are no 4's, and the 3 is in the wrong position)

YOU: 5153

ME: Fermi pico! (Now we know the right position of 3 is in the spot we haven't tried yet, and that there are no 5's)

YOU: 3166

ME: Triple fermi! (We're getting close! One of the 6's is in the correct position.)

YOU: 3167

ME: Double fermi pico!(The 6 is in the wrong position, and there is no 7 in this number.)

YOU: 3186

ME: Quadruple Fermi! (You got it!)

Exercises

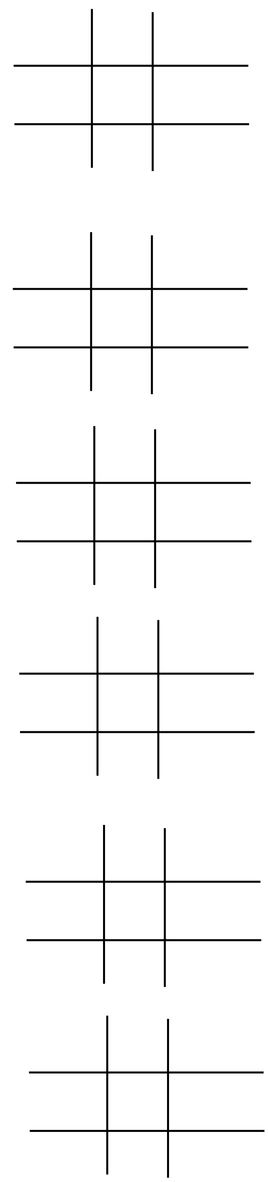
Your exercise for this lesson is to play this game (start with two-digit numbers) until you can see a pattern to quickly guess any number. Have fun!

Lesson #26: Tic-Tac-Toe

Overview: The first folks to play this game lived in the Roman Empire, but it was called *Terni Lapilli* and instead of having any number of pieces (X or O), each player only had three, so they had to move them around to keep playing. Historians have found the hatch grid marks all over Rome. They have also found them in Egypt!

In 1864, the British called it "noughts and crosses," and it was considered a "children's game," since they would play it on their slates. In recent times (1952), OXO was one of the first known video games, as the computer played games against a person.

Tic-tac-toe can be fun, but when you get a "cat's game" (no winner), it can get a little boring pretty quickly, right? I'll show you some cool ways to change the game to make it more interesting by changing one or two of the basic rules. It's much more engaging and strategic that way! Currently there are more than 100 variations of tic-tac-toe, and I'm going to show you my favorite ones. In fact, last time I taught a live science workshop, all 120 kids played this at the same time with squeals of delight!



Materials

- Pencil
- Paper

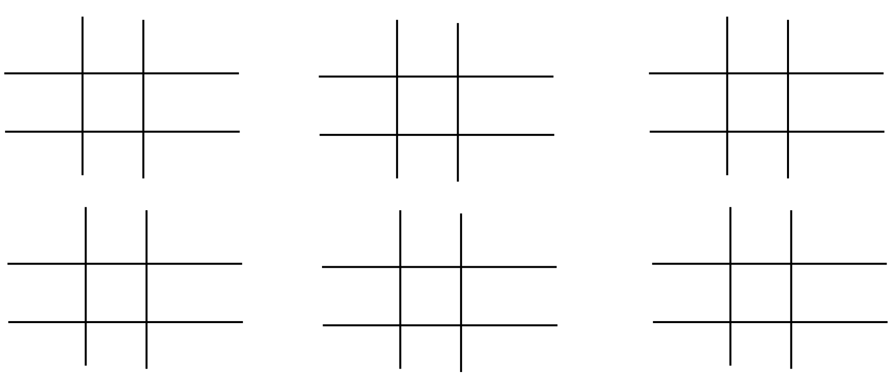
Activity: What happens in most tic-tac-toe games? Let's find out.

Step 1: Play one right now in one of the grids on the right, and then come back for step 2. Go ahead!

Did you find that there wasn't a winner? A tie game?

Okay, so let's change it a little to make it more interesting.

Step 2: Play another game, but this time on your turn, you can place an X or an O, and you can change your mind from turn to turn. And so can your opponent. The winner is the one who finishes any row, column, or diagonal with all X's or all O's. Go ahead and play a few games now!



Step 3: Did you like that last version? If so, then you might like this one! Instead of X and O, use numbers 1-9, and each number can only be used once. Take turns writing down the number in the grid. The first person to get to 15 wins!

The first round

		9
8		

The second round

		9
8		6
	7	

The third round

	5	**9**
8		6
4	**7**	

Fourth round

	5	**9**
8	**3**	6
4	**7**	

In the fourth round, the first player can place a 3 in the cell at the center to win the game.

You can add an extra rule that says you have to start with 9 and work your way sequentially down to 1, so it is easier to keep track of the numbers.

Step 4: **"**Last one wins." Start a new game, and on your turn, you can mark as many spaces as you want in the same row or column (no diagonals in this version). The person who marks the last space on the board is the winner.

Step 5: If you have lots of people who want to play, then this larger version of 4-in-a-row is perfect. (It's best with 3 to 6 players.)

					R	R			
		B					A		
			C	C					
						B			
		X		O					

First draw a grid that is 10 by 10. In this example game, we have 5 players (B, R, A, X, C, O). On their turn, they mark their box with a letter (you can also use shapes or simple pictures like a happy face). First person to get four in a row wins!

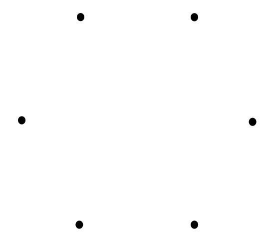
Which variation of tic-tac-toe did you like best? Do you think it changes the game a lot to let each player choose either X or O during their turn? The number tic-tac-toe game is pretty neat – I've found it's easier to keep track by starting with 9 and going down to 1, alternating turns and numbers as you go. Last One Wins is great, too, but 4-In-A-Row Tic-Tac-Toe may be my favorite. It's really fun for the whole family to play!

Lesson #27: Don’t Make a Triangle

Overview: This is a cool two-player geometry game with lots of strategy involved. You’ll need paper and two different-colored markers or crayons. The object is *not* to draw a triangle (or to force your opponent to draw one).

Materials

- Two different colors of crayons, markers, or pens
- Paper

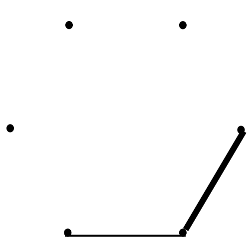


Activity: To start off, make the board by drawing six dots to form the corners of a hexagon. (I’ll draw lines of different thicknesses instead of having different colors.) You have the thicker pen, and I have the thin one. Here’s how it works:

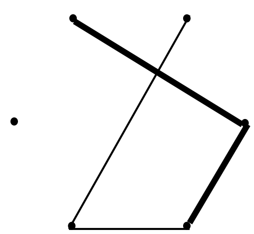
The goal is to connect the dots *without* making a triangle (of any size) with your own ink color. You may make a triangle using more than one color. Look at the diagrams to see what I mean.

I’ll go first with the thin line on the first turn. Then you draw the thicker line.

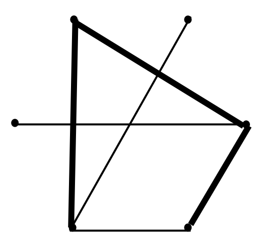
We’ll do this for a couple of rounds, but then do you see what happens on turn #7? The move MUST be a triangle!



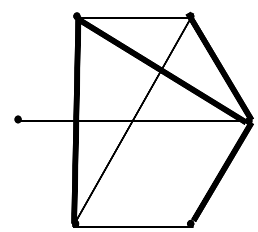
1st turn



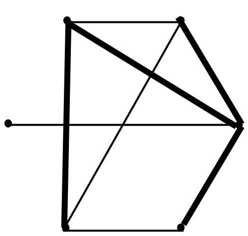
2nd turn



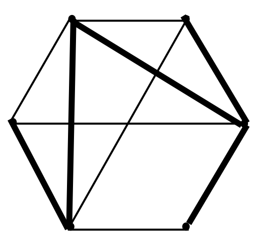
3rd turn



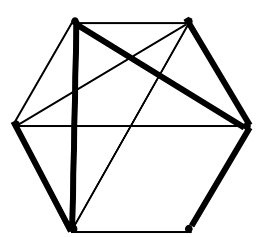
4th turn



5th turn

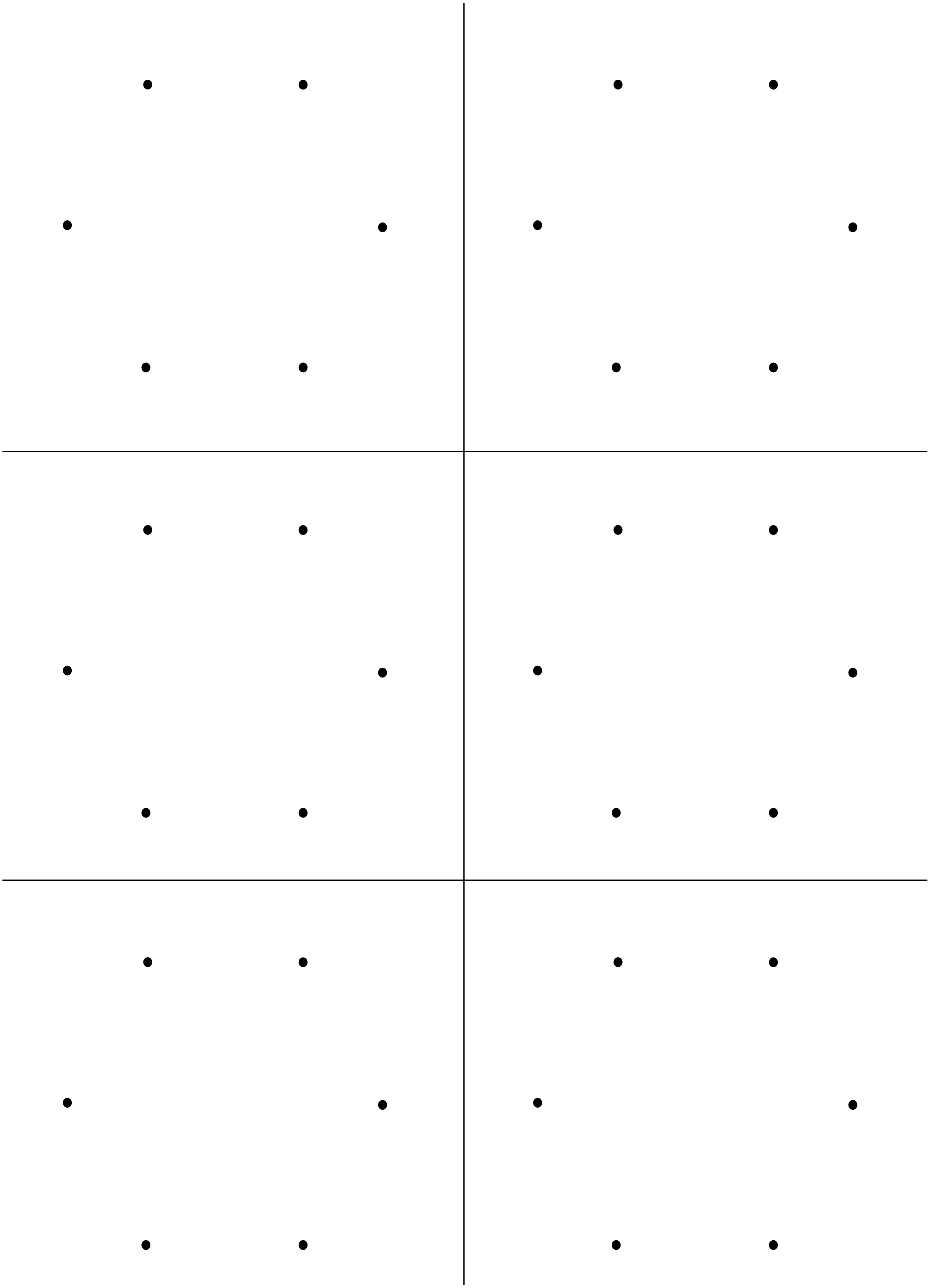


6th turn



7th turn

Now you try! Turn the page for a few pre-made hexagons to try this out on.

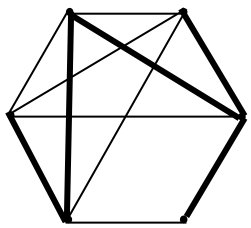


Exercises

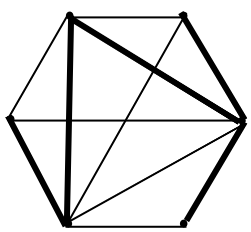
1. What is a triangle?
2. What is the name of the largest figure that can be generated when the six dots in the lesson are connected to each other?

How many triangles would the player with the faint pen make in the figures below?

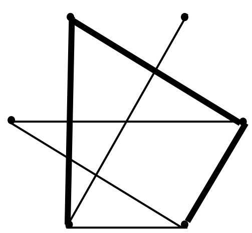
3.



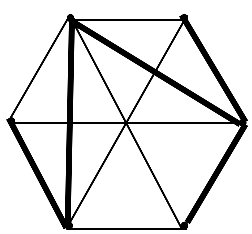
4.



5.

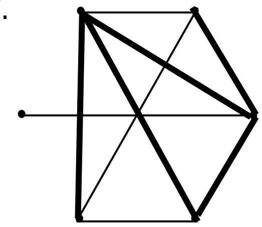


6.

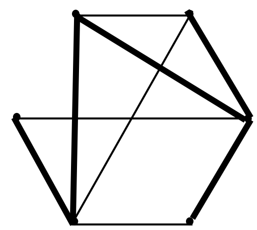


What is wrong with the player with the thick ink pen making the following moves (If there are triangles made, identify the number?)

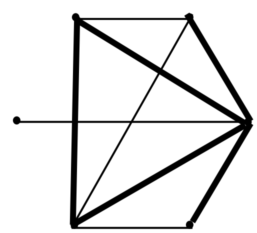
7.



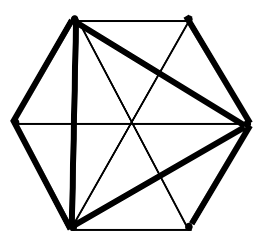
8.



9.



10. Identify the name of the largest quadrilateral made by the player with the thick ink pen in the diagram below.



Lesson #28: Math Dice game

Overview: Did you know I carry a set of dice in my pocket just for this game? It's as old as the hills and just as fun to play now as it was when I was a little math whiz back in second grade. (No kidding – when we had "math races," I was always team captain. Not quite the same thing as captain on the soccer field, though…)

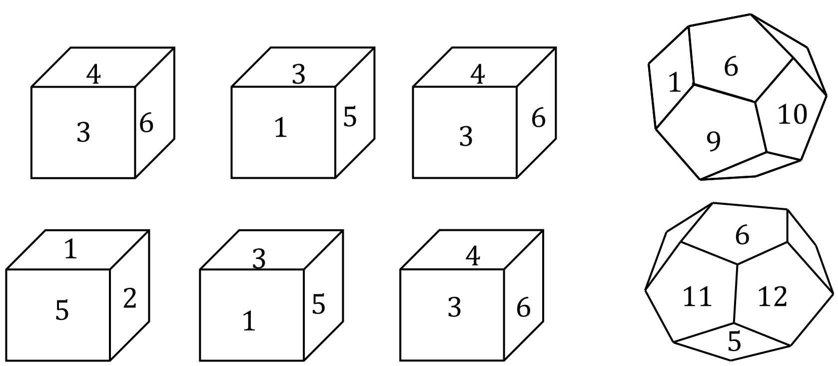
This is one of those quick-yet-satisfying dice games you can play to hone your thinking skills and keep your kids busy until the waiter arrives with your food. All you need are five or six standard 6-sided dice and two 12-sided dice. (Note – if you can't find the 12-sided dice, just skip it for now. You can easily substitute your brain for the 12-sided dice. I'll show you how.)

You need to do two things to play this game. You can use a calculator with this game, but usually the kids *without* the calculator win the round, as it usually takes *longer* to punch in the numbers than figure out the different possibilities in your head (and that's what you're working on, anyway!) If you're a highly visual learner, then use paper and a pencil so you can scribble down stuff as you go.

Materials

- Six 6-sided dice
- Two 12-sided dice
- Pencil (optional)
- Paper (optional)

Activity Lay your dice on the table and take a look at the possibilities. Six of them are numbered 1-6, and two are numbered 1-12. We use them in different ways. Here's how:



First, roll the two 12-sided dice. Multiply the two numbers in your head. If you rolled a 6 and 6, you'd now have 36. That's your target number. If you don't have the 12-sided dice, just think up two numbers, each between 1 and 12 and use those.

Next, roll the 6-sided dice. Now, using your arithmetic skills, figure out a way to add, subtract, multiply or divide those three numbers to get the number you rolled with the first set.

So if you rolled six 6-sided dice and got 4, 5, 2, 1, 1, 5, you could do this:

BOLD numbers are the ones rolled on 6-sided dice:

(**4** x **5**) – **2** = 18 18 x (**1** + **1**) = 36

You don't have to use all the numbers, but you can't double up and use a 4 twice if you only rolled one 4. And that's it! It's loads of fun and engaging, because it's got more than one answer. And sometimes there's ***no*** answer (although rarely!), which is just as fun to discover as one of the possible answers. As you get faster and better at this game, try taking away one or two of the dice. It's more challenging!

HOT TIP: You can add in the use of exponents when your kids get the hang of the game. An exponent tells you how many times to multiply a number by itself. For example, a 2 with a 3 exponent looks like:

$2^3 = 2 \times 2 \times 2 = 8$

To use exponents, simply use one of the numbers as an exponent. For example, if you rolled five 6-sided dice and got 5, 2, 3, 2, 4, you can do this to get 36 (imagine that the "36" came from the two 12-sided dice):

BOLD numbers are the ones rolled on 6-sided dice.

(**5** x **2**) = 10^2 = 100 100 – ($\mathbf{4^3}$) = 36

When your kids get good, you can up the ante and use a set similar to what I have in my purse: one 20-sided "double die" (which is a clear plastic 20-sided die that encloses a smaller, solid 20-sided die so it's two 20-sided dice in one package) and a handful of 6-sided. Math craze, anyone?

Exercises

No dice? Try these combinations that I rolled and recorded for you. I rolled six 6-sided dice and two 12-sided dice. Can you figure out a combination that would make each target number? Remember that the target number is the product (multiplication) of the two numbers from the 12-sided dice.

	<u>6-sided dice</u>		<u>12-sided dice</u>
1.	2,3,5,6,1,3	---	10,2
2.	4,3,6,4,5,3	---	12,9
3.	1,6,4,4,2,1	---	5,8
4.	4,1,1,1,2,3	---	3,9
5.	1,6,6,5,5,1	---	9,7
6.	2,1,4,5,2,2	---	6,3
7.	3,1,6,5,4,1	---	11,4
8.	4,3,6,4,5,3	---	12,7
9.	2,2,3,3,4,6	---	4,6
10.	3,5,6,1,3,3	---	6,9

Lesson #29: Big numbers

Overview Numbers really can be huge – some are too big to even imagine! Have you ever seen a million pieces of candy? Or have you ever even tried to count to one million? We'll try to figure out about how long it would take just to count to one million. I'll also show you how to write some really big numbers!

Materials

- Pencil
- Paper

Activity: When I was a math student in fourth grade, a friend of mine came over and told me she could prove that there was no such thing as a big number. I was interested, because I was *sure* there was (I mean, a million sure seemed big to me). So she asked me to think of any number. I came up with 8.

Then she asked if "1" was a big number.

I said no.

"Great," she said. "Now add 1 to your 8. What do you get?"

"Nine," I said.

"Is 9 a big number?"

"No," I said.

"Okay, now add 1 to 9. What do you get now?"

"10."

"What if you add 1 to 10?"

I wasn't seeing where this was going. "11," I said.

"So," she said carefully, "if you can just creep up on the numbers, like we did by adding 1 to the previous number like we did with the 9, 10 and 11, and each one isn't really that big after we add 1, then there's no such thing as a big number."

Her look was so triumphant I couldn't think of what to say. But I *knew* something wasn't right about her logic. According to her thinking, there could never be a 'big number' because it was only one unit larger than the previous one.

At what point does a number become a "big number?"

Her thinking was logical, but it wasn't right. Can you figure it out?

How big is big? Have you ever counted to a million? I know I tried. I didn't get very far!

Can you imagine a million candy bars? How about a million pennies? A million sheets of toilet paper?

A million is a big number, but compared to what? If you had a book with a million words, it would be larger than any book you've ever held. In fact, a book can never even have a half million words. The secret behind whether a number is large or not is what it's compared to. The number 1 can seem gigantic if you're measuring UV light wave frequencies, whereas 1 million can be teensy if you're measuring the miles between stars. So always ask: compared to what?

Okay, so let's imagine that we really wanted to count to a million. How long would it take?

There are 60 seconds in one minute: $\frac{60\ sec}{1 min}$

There are 60 minutes in one hour: $\frac{60sec}{1\ min} \times \frac{60min}{1\ hr} = \frac{3600sec}{1hr}$

There are 24 hours in each day: $\frac{3600sec}{1\ hr} \times \frac{24hr}{1\ day} = \frac{86{,}400sec}{1day}$

And there are 365 days in a year: $\frac{86{,}400sec}{1\ day} \times \frac{365days}{1\ yr} = \frac{31{,}536{,}000sec}{1yr}$

There are 31,536,000 seconds in a year. But you couldn't spend all of your time counting (you'll need to eat, sleep and use the bathroom!) And larger numbers take much longer than a second to say out loud. Taking all this into account, it would take more than a year to count to a million, and most people get bored with counting way before that.

Exercises

1. How many seconds are there in one hour?
2. How many seconds are there in a day?
3. How many days does a leap year have?
4. Write a number greater than but closer to a billion.(Note the difference between the two should be less than 50)
5. Write a number lesser than but closer to a billion.(Note the difference between the two should be less than 50)

Write the following numbers out numerically:

6. A thousand billion
7. A thousand million
8. A hundred hundred
9. Write a number that is 1 less than 100,000,000,000

Lesson #30: Exponents

Overview: Scientists use exponents all the time to write very large or very small numbers in a very short space. It's like shorthand for long numbers. It's also easier to work with large numbers in exponential form.

Materials

- Pencil
- Paper

Activity: Can you write a hundred thousand?____

To write such a number, we have to know the number of zeros a hundred has, and how many zeros you'll find after the 1 in one thousand.

One hundred = 100 (2 zeros)

One thousand = 1,000 (3 zeros)

So one hundred thousand is a 1 with 2 + 3 = 5 zeros after the 1: 100,000. To write this number in exponential form, write a "10" with a "5" as a superscript like this: $100{,}000 = 10^5$

Here is another question: Would you rather have a million billion dollars, or a billion million dollars?

Let's take a look: a million billion is written like this:

1 million = 1,000,000 which is a 1 followed by six zeros = 10^6

1 billion = 1,000,000,000 which is a 1 followed by nine zeros = 10^9

1 million billion is a 1 followed by 6 + 9 = 15 zeros: 1,000,000,000,000,000 otherwise known as one quadrillion, and scientists write this as 10^{15}.

1 billion million is a 1 followed by 9 + 6 = 15 zeros, which is exactly the same as a million billion. So you'll take either one!

Exercises

Write out each number long-ways (with all the zeros written out):

1. A thousand million
2. A thousand billion
3. Ten million
4. A hundred billion

Write the exponential form (ten and a superscript) of the following numbers

5. A thousand million
6. A thousand billion
7. Ten million
8. A hundred billion

Determine the exponents of the following number if written in the form; ”ten and a superscript.”

9. 10,000,000,000
10. 100

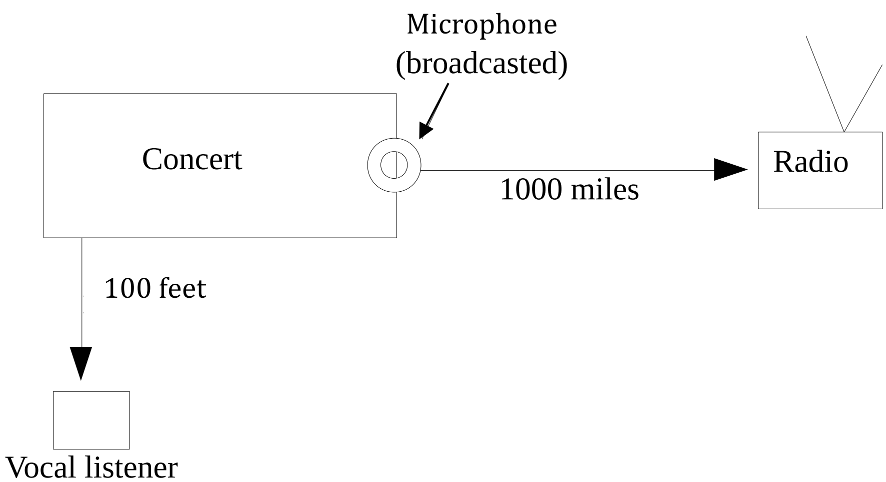
Lesson #31: Math at a Rock Concert

Overview: Here's an interesting math puzzle. If you're at a concert that's also being broadcast live on the radio, who will hear the music first? Will it be you, or people listening on the radio? I'll show you how to use the speed of sound versus the speed of light to find the answer.

Materials

- Pencil
- Paper

Activity: You've stood in long lines, paid for tickets, and now you're at a rock concert in the front row and get to hear it straight from the musicians themselves. This concert is so popular that they're also broadcasting it live to a radio station. The question is, do you get to hear the music first, or does the person listening on the radio?



The diagram above shows how things are set up: there's a live music performance at the concert. You're so close to the stage that you can hear their voices straight from their mouth. A listener on the radio is listening to a live broadcast from their radio at home. Let's figure out the difference between the time it takes the music to reach a listener 100 feet away via sound waves, and 1,000 miles away via electromagnetic waves.

Which do you think will be able to hear the music first?

As you make your guess, consider that 100 feet is a lot shorter than 1,000 miles! It seems obvious that the person who paid top dollar for those front row seats is going to get to hear it first.

But now consider this: Sound waves travel at around 750 miles per hour. That's the speed that the sound from the musicians travels at to get to you in the front row.

But what about the radio broadcast? Radios transmit electromagnetic waves, not sound waves. Electromagnetic waves travel much faster than sound waves: They speed along at 186,000 miles per second.

Write down your guess here: ____

Now let's figure this out:

Distance = 1,000 miles

Speed = 186,000 miles/sec

$$Time = \frac{Distance}{Time} = \frac{1{,}000\ miles}{186{,}000\ miles/sec} = \frac{1}{186} sec = 0.005\ 376\ sec$$

It takes 0.005 376 seconds for the music to go from the musician to the radio listener. That's fast!

Now let's take a look at the person in the front row.

Sounds travels at 750 mph, which is about 1,100 feet per second. The person in the front row is only 100 feet away.

$$Time = \frac{Distance}{Time} = \frac{100\ miles}{1{,}100\ miles/sec} = \frac{1}{11} sec = 0.090\ 91 sec$$

So it takes the sound 0.090 91 seconds to get to the listener in the front row.

The question is: which number is smaller? 0.053 76 seconds is less than 0.090 91 seconds, so the radio listener hears the music *before* the front-row person does!

Exercises

1. At what speed do electromagnetic waves travel?
2. What is the speed of sound though the air?
3. What is the relation between time, distance and speed?
4. A particle travels at a speed of 10 meters per second in the air for 20 seconds. Determine the distance that it covers.
5. Convert the answer from question #1 above to the units of minutes instead of seconds.
6. Convert the answer from question #2 above to the units of minutes instead of seconds.
7. A radio listener hears the news from her radio 12,000 miles away from the broadcasting center. Determine the time it takes to receive the sound.(Use the speed from #1 above)
8. A person attends a public rally at stand at 200 feet away from the stage so that they can hear the person's real voice without the use of speakers. Determine the time taken to hear the sound. (Use the speed from #2 above.)
9. In a phone conversation, it takes 1 second for a person to hear her friend from the other end. How far are these people from one another?
10. During a public lecture, a student at the back takes 0.1 seconds to hear what the lecturer is saying. What is the distance between the lecturer and the student?

Lesson #32: Rail Fence Cipher

Overview: Cryptography is the writing and decoding of secret messages, called ciphers. Now for governments, these secret ciphers are a matter of national security. They hire special cryptanalysts who work on these ciphers using cryptanalysis. The secret is that solving substitution ciphers can be pretty entertaining! Ciphers are published daily in newspapers everywhere. If you practice encoding and decoding ciphers, you too can become a really great cryptanalyst.

Materials

- Pencil
- Paper

Activity: In this lesson, let's take a look at how to code and decode the Rail Fence Cipher. Here's your original message: MEET ON THURSDAY

Now write one letter on the top row and one on the bottom row, alternating as you go so it looks like this:

```
M   E     O     T   U   S   A
  E   T     N     H   R   D   Y
```

Now group the letters in groups of 5 so that it's harder for a snooper to break it:

MEOTU SAETN HRDY

Notice that the last group of 5 only has four letters. Fill the last spot with an optional letter like a Q or Z:

MEOTU SAETN HRDYQ

That's the message you send to your friend! Don't forget to tell them how you encoded it so they can break it easily.

Once they've received your message, they need to decode it. Here's how to do it:

Count up the letters and separate them into two equal groups:

MEOTU SA   ETNHRDYQ

Place the first group on top of the second, and space the letters apart a little:

```
M   E     O     T   U   S   A
  E   T     N     H   R   D   Y  Q
```

Now use a zigzag pattern to pull the letters off one at a time, starting with the M at the top left. Follow it by the first letter on the bottom row, then the second letter on the top, then the second on the bottom, and so forth until you have the entire message.

MEET ON THURSDAY Q   (You can ignore the last letter Q.)

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

Write the cipher of the following message:

1. COME TOMORROW AT 2PM
2. HE HAS ARRIVED AT THE AIRPORT
3. HE IS CARRYING A BAG
4. THEY ARE HERE NOW
5. COME AT MY PLACE

Write the cipher of the following message (use a three row zigzag)

6. COME TOMORROW IN MY OFFICE
7. LEAVE THEM BEHIND
8. WE WILL BE THERE SOON
9. The cipher below was encoded using a two rows zigzag method with six letters in every row. Decode it.
   IMOIG OACMN NW
10. The cipher below was encoded using a three rows zigzag method with four letters in every row. Decode it.
    ICINA ONOMM GW

Lesson #33: Twisted Path Cipher

Overview: This cipher method uses a matrix and a path in order to encode your message. The shape of the path you create within the matrix of a Twisted Path Cipher determines how difficult it will be to break the code.

Materials

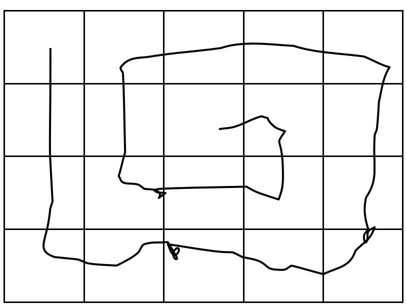
- Pencil
- Paper

I	N	S	I	D
E	T	H	E	O
L	D	P	I	A
N	O	X	Q	Z

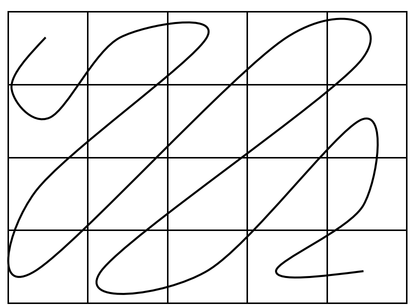
Activity: Let's code the message: "INSIDE THE OLD PIANO"

First count the number of letters in the message, then come up with a matrix (a table) that contains all the letters. Since this message has 17 letters, we'll need a 4 x 5 table as shown. Write one letter in each box, and if you have any empty boxes left, add a couple of extras like X, Q, and Z. (Tip: It's better to make a table with five boxes per row – you'll see why in a minute).

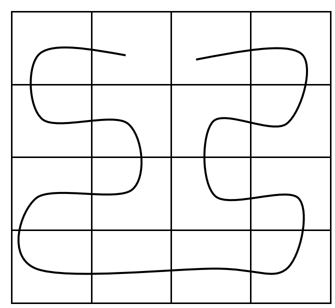
Now we choose a path in which we'll be listing the letters to form the cipher. You can choose a spiral in clockwise direction and anticlockwise spiral, a wavy line, or anything else you can dream up. Just make sure that the line goes through *every* box, even the extra letter boxes. And no breaks in the path – it's got to be a continuous line.



Spiral

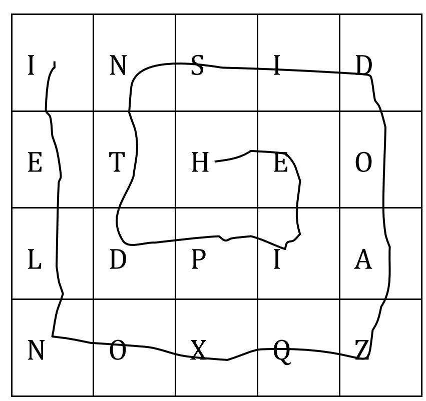


Wavy line



Series of connected wavy lines

Let's use a clockwise spiral. Begin at H and write the letters that lie along the path, grouping them in groups of 5 as you go along using this diagram:



HEIPD TNSID OAZQX ONLEI

When you give your friend this message, make sure you also let them know what kind of path you chose so they can break it easily!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

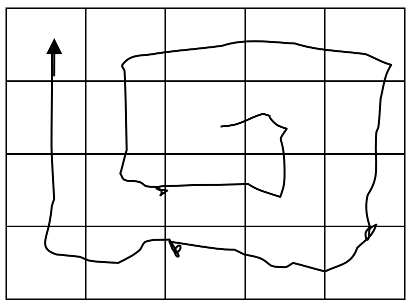
Exercises

Given the following message, what would be the best size of the table to use?

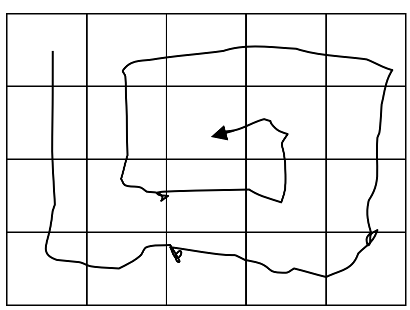
1. COME TOMORROW
2. WE ARE WAITING AT THE BUS STATION
3. THE LETTER IS IN HIS BRIEFCASE
4. HE WILL COME LATE TODAY

Identify the type of path indicated in the diagrams below

5.



6.



Write the cipher of the following message using the path in #5 above.

7. I WILL LEAVE FOR UK TODAY
8. AM ON MY WAY TO THE SCENE
9. I WILL ATTEND THE OPENNING DAY
10. I WILL BE THERE AT 2

Lesson #34: Shift Cipher

Overview: Shift ciphers were used by Julius Caesar in Roman times. The key is a number which tells you how many letters you'll shift the alphabet. These are fairly simple to encode and decode. However, you have to be extra careful when encoding because mistakes can throw off the decoding process.

Materials

- Pencil
- Paper

Activity: First you need a shift key. Let's pick a number: 4. (You'll give your friend the shift key when you also give them the message so that they can decode it easily.) This means that the alphabet will shift by 4 units. For instance, A will be shifted to W; J will be shifted to F and so on. The table below shows two alphabets: The top is the original alphabet before the shift, and the bottom is the one you'll use when you write the message.

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

Now come up with a message. I've got one: "BEN HAS BIG TOES"

To encode the message, find the "B" on the top alphabet, drop down and find the shifted letter under the "B." Did you find an "X?" Perfect.

What letter is going to be replacing the "E?" Find "A."

"N" is replaced with a "J"… and so forth until you encode the entire message to read:

XAJ DWO XEC PKAO

But remember, we like to group the letters into groups of 5 to throw off snoopers from our trail, so make the message look more like this:

XAJDW OXECP KAOXZ

In order to get the message decoded, give your buddy the shift key and stand back.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

Use the key shift number 4 on the letters of alphabet to code or decode the following:

Encode the following statement:

1. OUR SCHOOL WAS THE BEST
2. I WAS THE BEST IN MATHEMATICS
3. I AM THREE FEET TALL
4. OUR SCHOOL HAS GOOD TEACHERS
5. COME TO MY RESCUE

Decode the following:

6. SAWNA HAWRE JCXQZ
7. SDWQE OQDAM PAOQE KJXQZ
8. LHAWO AYKIA QKZWU
9. EWIKJ IUSWU
10. ODAEO WCKKZ OEJCA NMXQZ

Lesson #35: Date Shift Cipher

Overview: The Date Shift cipher is a much harder code to break than the simpler Shift Cipher. This is because the Date Shift number key varies from letter to letter, and also because it's *polyalphabetic* (this means that a number or letter can represent multiple letters).

Materials

- Pencil
- Paper

Activity: First, you need a date. I like to use the date that the message was sent. Suppose we pick a date: May 12, 1996. Let's figure out the shift key based on the date: change the date into a numerical date like this: 05/12/96. Now erase the slashes and write all the numbers together like this: 051296. That's our shift key. Let me show you how to use it.

I have a message to code: "LOOK UNDER DESK"

To encode it, write the date shift key 051296 above the message. If the message is longer than the key, just repeat the number sequence of the key to fit the message.

0 5 1 2   9 6 0 5 1   2 9 6 0

L O O K   U N D E R   D E S K

The numbers above the letters represent the shift key for that letter. All letters are shifted by their own personal shift key, like this:

L is not shifted at all, since its shift number is zero, so L = L.

The first O will be shifted 5, so it becomes a T.

The next O will be shifted once to become a P.

K is shifted by 2 to become M.

U is shifted 9 times which becomes a D. (You'll have to start at A after you get to Z to make this work.)

When you shift all the letters and group them in groups of 5, your message becomes:

LTPMD SDJSF NYK

Fill in the last few spaces with holders: ZQ.

LTPMD SDJSF NYKZQ

To decode the message, make sure you give your friend the date shift key, or it will be nearly impossible to break this code (even if they're good at breaking substitution ciphers).

Here's how to decode the message:

0 5 1 2   9 6 0 5 1   2 9 6   0 5 1

L T P M   D S D J S   F N Y   K Z Q

Now shift each of the letters backward:

L is not shifted, so it stays as L.

T is shifted 5 times backward to become an O.

P is shifted back one space to become an O. (Notice that both T and P represent O. That's why regular substation cipher decoding doesn't work!)

Continue shifting back to decode and find the original message: LOOK UNDER DESK (Ignore the last "ZQ" term.)

What do you think about the Date Shift cipher? The possibilities for numerical keys are endless. You can use the date you're sending the message, birth dates, phone numbers, and more! Just remember to start decoding by writing the key numbers over the top of the encoded cipher. And *always* makes sure the decoder has the correct numerical key!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. Which kind of key is used in date-shift ciphers?
2. In which direction is the cipher shifted when decoding?
3. How do you describe a cipher where a specific alphabetical letter represents more than one letter?

What would be the date shift key codes for the following?

4. April 4th 1998 (Don't forget that the key needs six digits! Use "04" for the date.)
5. Jan. 28th 2012
6. Nov. 30th 2011

Encode the following:

7. COME TO MY HOME : March 16th 1999
8. THEY HAVE JUST LEFT : June 1st 2001

What are the original messages? (These messages were sent on July 16th 1992)

9. GLBZC GNHNQ
10. MLFZT GAATO GRMZQ

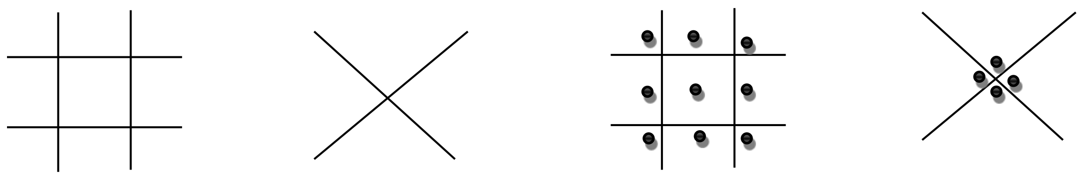
Lesson #36: Pig Pen Cipher

Overview: The Pig Pen cipher is of the most historically popular ciphers. It was used by Freemasons a century ago and also by Confederate soldiers during the Civil War. Since it's so popular, it's not a very good choice for top-secret messages. Lots of people know how to use this one! It starts with shapes: tic-tac-toe grids and X shapes. I really like it because coded messages look like they're written in an entirely different language!

Materials

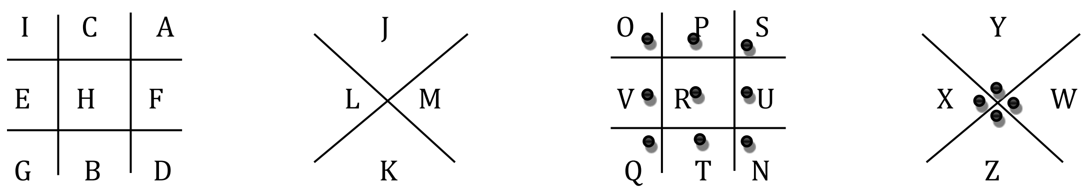
- Pencil
- Paper

Activity: First, you'll need to make the encoding key for your message. You'll need these four images to start with:



1st pig pen 2nd pig pen 3rd pig pen 4th pig pen

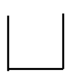
Now place one letter in each "pen" like this:

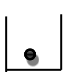


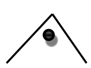
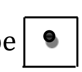
1st pig pen 2nd pig pen 3rd pig pen 4th pig pen

You can put any letters in any pen – the more random you are, the harder it is to break.

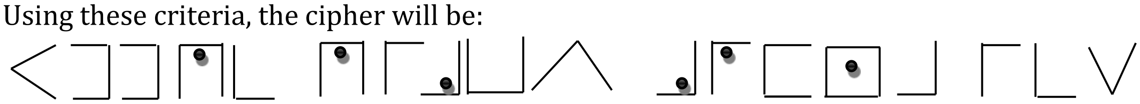
I've got a message here: "MEET AT DOCK ON FRIDAY"

For instance C is enclosed by  , that would be the symbol for C

P is enclosed by , that would be the symbol for P

The symbol for Z is  while that for R will be 

Using these criteria, the cipher will be:



Make sure your friend knows the key when you give them the code!

Now it’s your turn! Work out the exercises below. (You’ll find answers at the back of this book.)

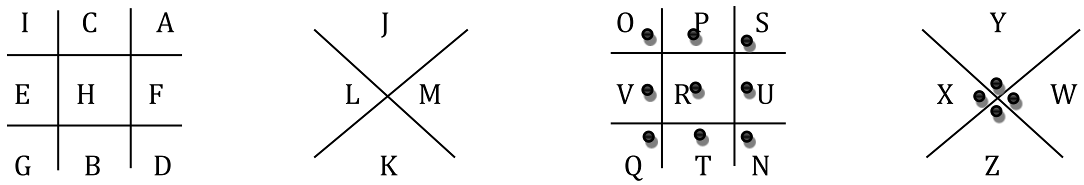
Exercises

1. What’s the difference between the second and the fourth pig pen?

Draw the following:

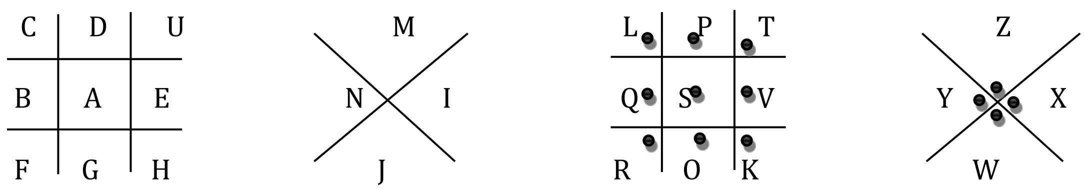
2. The third pig pen
3. The first pig pen

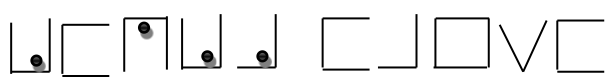
Using the following pig pen key, encode #4-7:

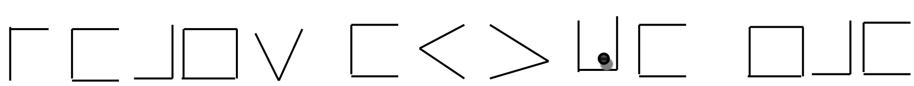


4. GO TO SCHOOL
5. COME BACK
6. I WILL SLEEP LATER
7. NORTHERN AMERICA

Using the pig pen key, decode the following messages:



8. 

9. 

Lesson #37: Polybius Checkerboard Cipher

Overview: Polybius was an ancient Greek who first figured out a way to substitute different two-digit numbers for each letter. In the Polybius Cipher we'll use a 5×5 square grid with the columns and rows numbered.

Materials

- Pencil
- Paper

Activity: To make the key, we need a 5x5 grid. We'll let Y and Z share the last space, as Z isn't used much. Then write one letter in each box like this:

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y/Z

To substitute for a letter, we use the number of the row and the column. Remember the order is first the row, then the column.

For example:

L is in row 3 and column 2. So in place of L you'd write: "32"

I is in row 2 and column 4, so it's replaced by "24"

X is in row 5 and column 4, so you'd write "34"

Now let's encode the message: "STOPWATCH" like this:

44-45-35-41-53-11-13-23

To decode the message, we follow the same thing in reverse:

44 means look at the 4th row, 4th column to find S

53 means look at 5th row, 3rd column to get W

Continue until you get all the letters to read: "STOPWATCH"

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. Write the grid that is used to encode and decode messages in the Polybius Cipher. (Can you think up your own?)
2. Identify the codes representing the following letters using the grid from the example in the lesson: A, O and C.

Encode the following statements using the Polybius Cipher grid from the main lesson:

3. LET US LEAVE
4. I AM LEAVING
5. THEY WILL COME
6. SHE IS HERE

Decode the following messages using the Polybius Cipher grid from the main lesson.

7. 32-35-35-31-11-45-45-23-15-45-11-12-32-15
8. 24-45-24-44-23-24-14-14-15-34
9. 23-15-43-15-24-44-45-23-15-41-11-41-15-43
10. 32-35-35-31-11-45-33-55-12-35-35-31

Lesson #38: Cracking Ciphers

Overview: Cryptograms are solved by making good guesses and testing them to see if the results make sense. Through a process of trial and error, you can usually figure out the answer. Knowing some facts about the English language can help you to solve a simple substitution cipher. For example, did you know that an E is the most commonly-used letter in the English alphabet? It's also the most commonly-used letter to end a word.

Materials

- Pencil
- Paper

Activity: It helps to have a lot of words to work with so that you can begin to recognize patterns in the code. Here are some examples:

Single-letter words will most likely be either **A** or **I**

The most frequent two-letter words in English are **OF**, **TO,** and **IN.**

The most frequently used three-letter words are **THE** and **AND.**

The most commonly used letter in English is E, then T, A, O, N.

The most common letter that appears at the end of words is E.

The most common letter that appears at the beginning of words is T.

Finally, the most frequently occurring four-letter word in English is **THAT.**

Additional tips include the fact that **Q** is almost always followed by **U**, and that **N** often (but not always) follows a vowel. Finally, if two code symbols occur in a row, they could be a consonant combination such as **LL, EE, SS, OO, TT**, etc.

The real trick to this is to try something, and then if it doesn't work, go back and try something else. Get lots of practice by checking in newspapers and magazines for these popular puzzles. If you really like them, you can find puzzle books full of cryptograms. You'll be an expert cipher solver in no time!

Let's break the code *without* using a key:

ZU HO UD CUZ ZU HO ZSGZ AF ZSO JKOEZAUC

To begin with, look at the words that seem to form certain patterns. Do you notice the two-letter word “HO” and “ZU?” We can’t tell yet what they are, so let’s keep looking for something we can identify.

Check out the four-letter word “ZSGZ.”. The first and the last letter “Z”, and my bet is that the word is “THAT.” If so, we already know three letters! Let’s try it:

```
ZU HO UD CUZ ZU HO ZSGZ AF ZSO JKOEZAUC
T          T T     THAT    TH      T
```

Now look at “ZSO.” What word do you think it is? My guess is “THE,” so now O = E. Replace all O’s with E’s:

```
ZU HO UD CUZ ZU HO ZSGZ AF ZSO JKOEZAUC
T   E      T T   E THAT    THE   E T
```

Check out “ZU.” What word do you think it is? What about “TO?” Let’s substitute U = O:

```
ZU HO UD CUZ ZU HO ZSGZ AF ZSO JKOEZAUC
TO  E O    T TO  E THAT I  THE   E T O
```

Look at the last word: “J K O E Z A U C.” What do you think that word is? Notice the T_O_ at the end of the word. My guess is that those last four letters are “TION.” So Z = I and C = N:

```
ZU HO UD CUZ ZU HO ZSGZ AF ZSO JKOEZAUC
TO  E O  NOT TO  E THAT I  THE   E TION
```

If so, then you can guess at the word – my guess is that it’s “QUESTION.”

Try to read the sentence from time to time to see if you can figure it out without having to know all the letters first. I read it over, and this popped into my mind:

TO BE OR NOT TO BE THAT IS THE QUESTION.   Which also turns out to be the answer. ☺

Now it’s your turn! Work out the exercises below. (You’ll find answers at the back of this book.)

Exercises

1. What does it mean by “cracking a cipher?”
2. In there a difference between cracking and decoding a cipher?
3. What is very important that a person should know before beginning the cracking process?
4. What is the most common letter of alphabet that is usually at the end of a word?
5. What is the most common letter that is usually at the beginning of a word?
6. If you have a letter all by itself, what is it most likely to be?
7. What are two of the most common two character words in sentences?
8. What are two of the most common three character words in sentences?
9. What is the most common four character word in sentences?

Lesson #39: Playfair Cipher

Overview: This is a type of cipher that is very difficult to break because no specific letter or number represents any specific letter. The Playfair Cipher uses a matrix of numbers and letters to develop the key.

Materials

- Pencil
- Paper

A	H	M	V	L	3	Y	D
X	K	B	5	P	Z	E	O
N	7	W	U	F	T	6	J
G	R	2	Q	C	A	I	S

Activity: This is a super-hard cipher to break. It's encoded by taking pairs of letters and numbers from a matrix. There are three rules to follow:

If both letters are in the same row, then use the letters immediately to the right of each other. (Think of the rows as wrapping from the right end back around to that same row's left end).

If both letters are in the same column, then use the letters immediately below them. If necessary, the bottom letter wraps back around to the top of the same row.

If the two letters or numbers are in different rows **and** in different columns, then each letter is replaced by the letter in the same row that's also in the same column of the other letter. Basically, you find each intersection of the pair. Use the letter or number below the pair and then the one above the pair.

Play Fair sounds really complicated, but that also makes it a tough code to crack! Let's do an example:

I WILL ARRIVE AT FOUR PM

We group the message in twos:

IW IL LA RX RI VE AT FO UR PM

Note that we added an "X" between the double R term "RR," as we can't encode "RR" with this method.

Also note that the number of characters must be even, so add a space filler like Z or Q as needed.

To encode the message:

IW: These are in different rows and columns, so we have I–2 and W-6 to make "26."

LA: These are in the same row so we have L-3 and A-H to make "3H."

AT: These are in the same column so we have A-3 and T-A to make "3A."

Replacing the pairs of letters in the original message we now have:

26 CY 3H GK 25 5Y 3A JP 7Q BL

To make it even harder, you can group them in groups of four or five to get:

26CY 3CGK 255Y 3AJP 7QBL

Just remember, when *decoding* the Playfair Cipher, you have to shift *up* instead of down and *left* instead of right. And it's easy to make a mistake by encoding in the incorrect order. Always double check your cipher before sending it on to the recipient. Mistakes make messages much harder for the decoder to interpret.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. What is the name given to the following table?

A	H	M	V	L	3	Y	D
X	K	B	5	P	Z	E	O
N	7	W	U	F	T	6	J
G	R	2	Q	C	A	I	S

Use the table in 1 above to answer question 2 – 10.

What will be the cipher for the following?

2. KB
3. HR
4. AR
5. EU
6. COME TO SCHOOL
7. GO HOME THEN

Decode the following messages

8. 73 3N SG YZ 6X
9. SG MN YK A7 JO HD

10. Why is it important the number of letters in the message to be encoded be even?

Lesson #40: Telephone Cipher

Overview: This is a cipher that is written using the telephone keypad. It's pretty simple to use once you understand the basics of how to make it work for the entire alphabet. (Have you ever noticed that the Q and Z are missing from the keypad?)

Materials

- Pencil
- Paper
- Access to a telephone keypad, or use ours below

Activity: Code machines – or cipher machines – can be used to encode and decode messages. One everyday example of a code machine that you can easily access is a telephone.

Here's the dial pad (keypad) of a telephone with Q and Z added in:

Q	ABC	DEF
1	2	3
GHI	JKL	MNO
4	5	6
PRS	TUV	WXY
7	8	9
	Z	
	0	

Notice how the number 2 represents A, B, and C. To be able to tell which letter you really mean when you write a 2, you'll also use symbols above the letter like this:

\ for the first letter, like A on the number 2 key

| for the letter at the middle, like B on the number 2 key

/ for the letter as the end, like C on the number 2 key

The message "PICK UP FISH" will be written as follows:

\ / / | | \ / / / |
7 4 2 5 8   7 3 4 7 4

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

Encode the following using the telephone keypad:

1. COME BACK
2. LEAVE TODAY
3. HE ARRIVED
4. THEY WENT
5. HE IS COMING
6. REVEAL
7. GO AHEAD

Decode the following ciphers

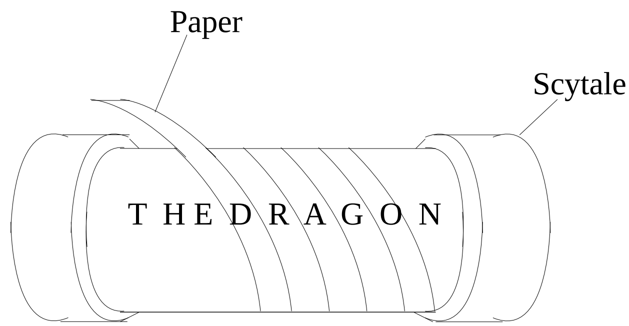
```
    \ \ | | /   / \
8.  1 6 3 7 4   2 2

    | | / \ |   \ | / | \   \ / \
9.  8 6 4 8 3   3 5 4 6 4   3 6 6

    \ | / \ |   \ / / \
10. 2 8 7 8 7   2 5 4 2
```

Lesson #41: Scytale

Overview: In this lesson, I'll show you how to use a actual cipher machine called a *scytale*. This was first used in ancient Greek and Roman times, most notably by the Spartans. To make a scytale, use a cylinder with a piece of paper wrapped around it. Then simply print your message in rows that run along the length of the cylinder. When the paper is unwrapped, the message is scrambled!

Materials

- Marker pen
- Paper strip (Make your own by cutting your paper into 1" strips and taping them together end-to-end.)
- Cylindrical object, like a toilet paper tube or paper towel tube



Activity: The Scytale machine is a cylindrical object that gets a long strip of paper wrapped around it. Start at one end and start wrapping the paper around the tube, keeping the edges of the paper lined up.

Now write your secret on the paper by placing one letter on each strip section as shown.

I have a message: "THE DRAGON EGG IS HIDDEN UNDER THE BUSHES".

After you write your message, unwrap the strip and you'll find a garbled message. The neat part of this type of cipher is that the receiver must not only get the strip, but also the correct diameter tube in order to decode the message.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. What is the name of the cryptographic machine that was first used by the ancient Greeks and in Roman times to send secret messages?
2. What is the shape of the machine named above?
3. Where is the message written when using this machine?
4. Apart from the machine, what else is required to be able to encode the message?
5. Who were the people who most notably used the type of cryptography named?
6. What is the major problem that the recipient must figure out to easily decode the message?
7. Is it true that first letter in the original message would be the first letter on the encoded message? Explain.

Lesson #42: Secret Phonetic Codes

Overview: If only you could keep better track of big numbers, adding and multiplying your head wouldn't be such a problem! But fear not… I have a trick that might be just the ticket for your brain!

Use this secret phonetic math code to code and decode sentences into numbers. Developed over a hundred years ago, this is *the* code that the expert mathematicians use when doing large calculations in their head. This is exactly how Dr. Arthur Benjamin from Harvey Mudd University squares 5-digit numbers in his head, *without* a calculator!

Materials

- Pencil
- Paper

Activity: The first thing you need to do is memorize this short list of number substitutions:

1 = t or d	6 = ch, sh, or j
2 = n	7 = hard c, k, or hard g
3 = m	8 = f or v
4 = r	9 = p or b
5 = l	0 = s or z

All vowels and letters like y and w are not used (you'll see what I mean in a minute). Let's try an example so you can see how this works:

To encode the number: 307

Find: 3 = m

0 = s or z

7 = hard c, k or hard g

Look at m, s, and k and try to make a word out of it by adding vowels. Looks a little like "music," doesn't it? Since vowels don't have a numerical value, 307 = music.

What if I give you "music"… can you decode it back into its original number? Try it now:

Let's memorize the first 24 digits of pi:

My turtle Pancho will, my love, pick up my new mover Ginger.

Use the phonetic code to write out the 24 digits of pi here:

Here's a hint: m=3, y has no value, t =1, u has no value, r =4, t = 1, l = 5, p = 5… and onward to get:

pi =3.14159265358979323846264 *Ta-daa!*

How about the next 17 digits of pi? What kind of words can you make with these digits using the phonetic code?

3 3 8 3 2 7 9 5 0 2 8 8 4 1 9 7 1

Try it here:

When I assign them the alphabet codes, one possibility is this: *My mauve monkey plays in a favorite booklet*

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

What numbers are these?

1. place
2. now
3. healing the disease
4. what is my effort
5. that he was not able to come
6. people will come and go

Form a phrase that represents the following numbers:

7. 6313406
8. 1071

Lesson #43: Factorials!

Overview: If I said "3!" would you think the 3 is really excited, or that you have to shout the number?

In fact, it's a mathematical operation called *factorials*, and boy, are they fun! They may seem complicated at first, but they're really a very basic concept. The exclamation point behind a number means that you multiply that number by each successively lower number, in order, until you get to 1. So "3!" would be equal to 3 x 2 x 1 = 6.

Materials

- Pencil
- Paper

Activity: Factorials are useful when you want to know the number of possible combinations or permutations that can be made from a set of objects. A permutation is when the order does matter, and a combination is when it doesn't matter (a permutation is an ordered combination).

Let's do a real-life example: Three friends go into an ice cream shop: Tom, Mary and Jean. Who orders their ice cream first? Second? Last? We can have lots of different ways in which these three people can order their ice cream.

Here are all the possibilities for the three people:

Tom, Mary, Jean

Tom, Jean, Mary

Jean, Mary, Tom

Jean, Tom, Mary

Mary, Jean, Tom

Mary, Tom, Jean

Did you notice how there are six possible arrangements for three people? This can be summarized as 3! = 6. What if there were four people? Then we would have 24 different arrangement styles since 4! =24.

Five friends? Then the answer is 5! = 5 x 4 x 3 x 2 x 1 = 5 x 24 =120.

Do you see how the factorial grows rapidly as you increase the number?

Let's take another example: a deck of 52 cards. The question is, how many different ways can the cards be arranged in the deck? 52! = 52 x 51 x 50 x ….x 3 x 2 x 1 = 80 million trillion trillion trillion trillion trillion

Can you see how factorials start to get really big, really quickly? The card deck is a really great example of this, because with 52 cards the factorial is 52!, which is a HUGE number. There are literally trillions and trillions and *trillions* of ways to arrange those cards. And *that's* why you'll probably never in your lifetime encounter a card deck where the cards are in exactly the same order more than once except when they first come out of the box.

Question: What does 2! = ?

Remember that 2! = 2 x 1 = 2

So then what is 1! =?   It's 1!

But what is 0! ?

Well, I can prove to you that 0! = 1 using the following logic:

$$6 = \frac{6!}{5!} = \frac{6\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}$$

$$5 = \frac{5!}{4!} = \frac{5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{4\ x\ 3\ x\ 2\ x\ 1}$$

$$4 = \frac{4!}{3!} = \frac{4\ x\ 3\ x\ 2\ x\ 1}{3\ x\ 2\ x\ 1}$$

$$3 = \frac{3!}{2!} = \frac{3\ x\ 2\ x\ 1}{2\ x\ 1}$$

$$2 = \frac{2!}{1!} = \frac{2\ x\ 1}{1}$$

Using the same pattern we will have:

$$1 = \frac{1!}{0!}$$

But 1 is a whole number, so:

$$1 = \frac{1!}{0!} = \frac{1}{1}$$

For this to be true, then: 0! = 1

Does 0! = 1 make sense to you? If not, that's okay. Just memorize this fact and tuck it away for later. It will come in handy some day in algebra and maybe even for calculus!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 6!
2. $\frac{6!}{4!}$
3. How many ways can seven different cards arranged uniquely?
4. 0! x 4!
5. 3! x 4!
6. 1! x 5!
7. 2! x 0 x 6!
8. 2! x 4!
9. 3! x 2!
10. 5! x 0! x 1! x 2!

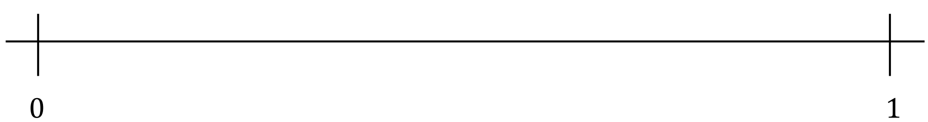
Lesson #44: Probability

Overview: In math, probability is how likely it is that something will occur (or not). Probability is expressed in a range from 0 to 1. A probability of zero means that a thing will definitely not happen – it's impossible. But a probability of one means that it definitely will happen – it's certain. Any number larger than 0, but smaller than 1, means that a thing might happen. The number 1/2, or one half, is right in the middle and it means there is a 50/50 chance. Do you think there's a greater chance for a person to get struck by lightning, or to be hit by a meteorite?

Materials

- Pencil
- Paper

Activity: Some key words that help with probability questions are OR and AND. When you see the word OR, it means you should be adding the possible outcomes to find out the probability whether one thing OR another will happen. The word AND means you will probably be multiplying to find the solution.



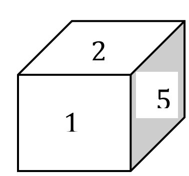
When something will not happen at all, then its probability of happening (the chances of it happening) is zero. When we are sure that something will definitely happen, the probability of occurrence is 1. When the probability is between 0 and 1, then something may or may not happen.

For example, if you flip a two-sided coin, you have two possibilities: a head or a tail side to show. The total number of possible outcomes is 2. Since only one side can be at the top, we have only one possible outcome. So the probability of a tails (or heads) showing after you flip it is:

$$\frac{Possible\ outcome}{Total\ possible\ outcomes} = \frac{1}{2}$$

Let's take an example. What is the probability that when a six-sided die is rolled, it will show a 1 or a 2?

Since a die has 6 faces, I expect only one face to show after I roll it, so all the faces have equal chances of showing up.



Using the formula above, the probability a 1 will show up is $\frac{1}{6}$, while the probability that 2 will show up is $\frac{1}{6}$.

Because the questions asked for the probability of a 1 OR a 2, we add them together: $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Let's try another example: what is the probability that a 6 and a 6 will show up in two successive rolls?

The probability that one 6 will show up is $\frac{1}{6}$, so the probability that a six *and* a six will show up is the product $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Since the question asked was a 6 *and* a six, the probabilities get multiplied together.

Just for reference: the probability that you will be struck by a meteorite is 1 in a trillion. The chance of being struck by lightning is 1 in ten million.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. What is the probability of a coin showing tails when flipped?
2. What is the probability of a coin showing heads twice in a row?
3. What is the probability that heads *or* tails will show up in a toss?
4. What is the probability that heads *and* tails will show up in two successive tosses?
5. A die is rolled once: what is the probability that a four will show up?
6. A die is rolled once: what is the probability that a three will show up?
7. A die is rolled once: what is the probability that a four *or* a six will show up?
8. A die is rolled twice: what is the probability that a four will show up in all the rolls?
9. A die is rolled twice: what is the probability that a four *and* a two will show up?
10. A die is rolled twice: what is the probability that the same number will show up in all the rolls?

Lesson #45: Three doors

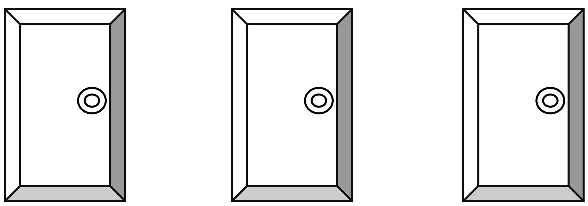
Overview: Ever dream about winning big on a game show? Would it surprise you to learn that there's math behind it all? Probably not, since you've made it this far through this book. Here's the deal: This lesson in probability teaches how to increase our chances of winning in a game.

Materials

Pencil

Paper

Activity: Imagine that you are on a game show with a chance to win a car. There are three doors and the car is behind one of them. You just have to choose the correct door! You can use probability to get a possible advantage in choosing the correct door.



You pick one of the doors at random. Remember that you have equal chances of opening a door of your choice. Since you only have one chance to open one of the three doors, the probability that you open the correct door is:

$$\frac{Number\ of\ chances}{Total\ available\ chances} = \frac{1}{3}$$

The game show host, who knows where the car is, opens a different door (not the one you chose) and shows you an empty room. Should you change your guess?

Mathematically speaking, would you increase your odds of winning if you switched to a different door?

Use the space here to figure out what you should do before moving on to the next page where the answer is.
(No peeking yet!)

Need a hint? Think of it this way: What if there were 100 doors? You pick one. You have a 1/100 chance of guessing correctly with your initial guess. The remaining doors combined together make up a 99/100 chance. The host, who knows where the car is and isn't going to show it to you, opens up 98 of the doors, none of which have the car behind it. The door you initially guessed is still a 1/100 chance, while the last unopened door is 99/100. You should switch your choice!

Not convinced? Imagine you chose door #1, which means you have a 1/3 chance of winning, and a 2/3 chance that the car is hidden behind one of the other doors. But the host gives you a clue by opening a losing door. Notice that if the car is behind door #2, the host will open #3, and if the car is behind #3, door #2 is opened. This is important because when you switch, you win if the car is behind #2 *or* #3, so you win either way! But if you don't switch your choice, you can only win if the car is behind door #1.

Still not convinced? The winning odds of 1/3 on the first choice don't increase to 1/2 just because the host opens up a losing door for you to see. The benefits of switching are really seen if you do this with three cups and a penny. Place a penny under one of the cups (make sure you don't use transparent water glasses!) and play six games that have all the possibilities explored.

You'll notice for the first three games, you choose cup #1 and then switch each time. For the second set of three games, you choose cup #1 and stay (don't switch your choice) each time. The host always reveals an empty cup to you. Here's what you'll find:

	Cup #1	Cup #2	Cup #3	Result
Game 1	Penny	None	None	Switch and you lose.
Game 2	None	Penny	None	Switch and you win!
Game 3	None	None	Penny	Switch and you win!
Game 4	Penny	None	None	Switch and you win!
Game 5	None	Penny	None	Switch and you lose.
Game 6	None	None	Penny	Switch and you lose.

Did you notice that you switch, you win 2/3 of the time, and only lose 1/3 of the time? And when you don't switch, you win 1/3 of the time and lose 2/3?

This is a classic problem that people get confused about. Thinking in terms of 100 doors makes it easier to see how the math works!

This problem and solution was originally published in PARADE magazine in 1990 and 1991.

Answers to Exercises

Lesson 1: The Magic of 11's

1. 121
2. 297
3. 273
4. 539
5. 550
6. 737
7. 869
8. 979
9. 1012
10. 1056

Lesson 2: Multiply 3 digit number by 11

1. 1793
2. 2585
3. 3795
4. 5269
5. 7249
6. 8228
7. 10967
8. 10802
9. 9603
10. 8459

Lesson 3: Multiply by 12's

1. 5,973
2. 540
3. 3,912
4. 9,228
5. 16,140
6. 41,532
7. 90,384
8. 107,868
9. 119,988
10. 1,184,988

Lesson 4: Divisibility

1. None
2. 3981,72624
3. 5894
4. Yes
5. 453970
6. It may or it may not
7. no
8. 2391
9. The numbers must be composed of 3 digits and their last digit must be 0 or 5
10. The numbers must be composed of 4 digits and their last two digit must be 00 or must form a number that is divisible by 4

Lesson 5: What day were you born on?

1. Wednesday
2. Wednesday
3. Saturday
4. Monday
5. Friday
6. Tuesday
7. Sunday
8. Friday
9. Lewis
10. Saturday

Lesson 6: 1 dollar word search

1. $1
2. $1
3. $1
4. 22 cents
5. Supper
6. Tuesday

Lesson 7:Isn't that SUM-thing?

1. 2233
2. 2986
3. 2000
4. 2997
5. 2425
6. 2775

7. 2557
8. 1998
9. 879
10. 567

Lesson 8: How to add and multiply fast in your head

1. 97
2. 217
3. 819
4. 6491
5. 1220
6. 4720
7. 16,400
8. 103,200
9. 11,950
10. 92,568

Lesson 9: Squaring Two-Digit Numbers

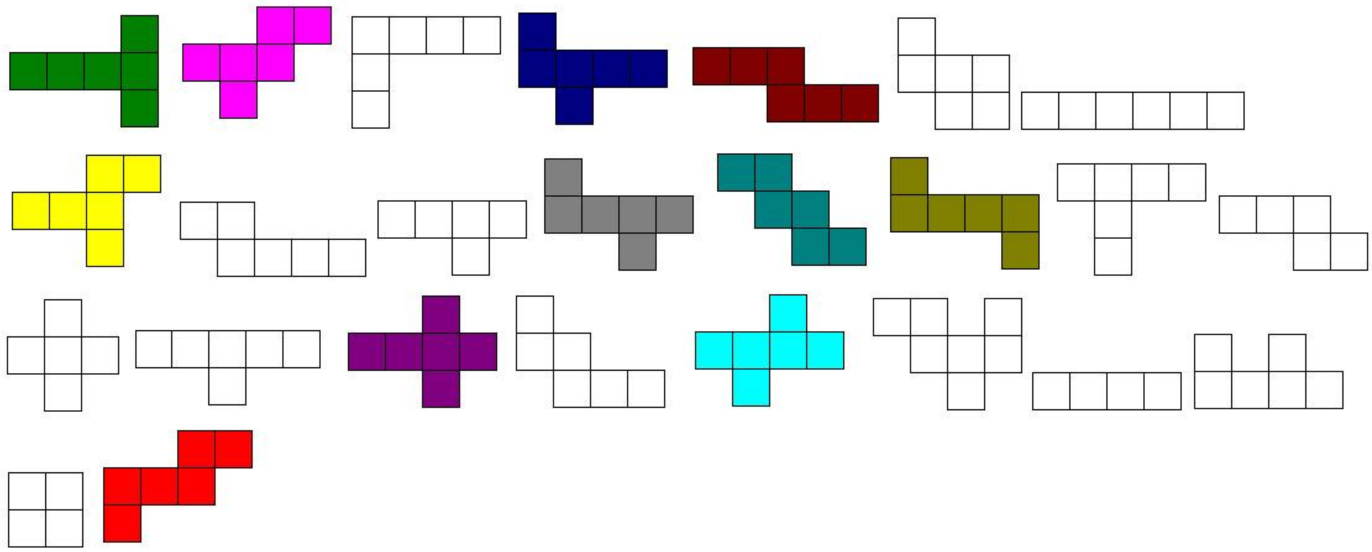
1. 441
2. 1849
3. 1369
4. 4761
5. 9801
6. 6724
7. 3364
8. 4096
9. 2809
10. 7396

Lesson 10: How to square bigger numbers

1. 8,649
2. 37,249
3. 958,441
4. 62,001
5. 172,225
6. 7,056
7. 328,329
8. 110,889
9. 573,049
10. 484,416

Lesson 11: Folding a Cube

1. A 3 dimensional object with six faces and 12 equal edges
2. 6



Lesson 12: Mobius strip

1. Paper strip
2. Circular strip
3. Möbius strip
4. Annulus
5. 1
6. 1
7. 180 degree
8. 1
9. None
10. 1

Lesson 13: Stepping through a sheet of paper

1. 4
2. ½
3. 1/16
4. ¼
5. 2
6. 4
7. To keep the strip connected (closed)
8. 8
9. 16

Lesson 15: Fractals

1. Fractals
2. A series of known geometrical figures such as triangles, squares and circles
3. Triangles
4. Equilateral triangles
5. 5
6. 8
7. Sierpinsiki triangle
8. Engineering field
9. Erase the middle third of the sides, draw the triangle at the very place and erase the inner lines
10. Divide the triangle into 3, ignore the middle part and repeat the step for the other small remaining parts

Lesson 16: Chaos Fractal Game

1. 3
2. 3
3. Before rolling the die for the first time
4. Triangles
5. At the vertices of the equilateral triangle
6. Die
7. 1/3
8. Each color painted on two faces of the die
9. Between the more recent dot and the player's first dot at the vertex of the triangle
10. Sierpinsiki triangle

Lesson 17: Real Geometry: The Pantograph

1. Pantograph
2. Scale down and enlarge drawings
3. Drill the holes where the screws will be fixed to hold the bars in position
4. Mount the sheet of paper on the drawing board
5. Drawing and pointing pencils
6. Larger than the original object
7. Smaller than the original object
8. It is fixed stationary
9. Enlarged to allow movement of the bars at the joints
10. Parallelogram

Lesson 18: Graphical Multiplication

1. 1035
2. 4032
3. 1352

4. 3038
5. 6164
6. 211 x 112 = 23 632
7. 234 x 313 = 73 242
8. 22 x 32 = 704
9. 64 x 53 = 3392
10. 24 x 31=744

Lesson 19: Paperclip trick

1. Is a number that can be divided by 2 completely without a remainder
2. Is a number that when divided by 2, the remainder is always 1
3. 3,(4(1))
4. 1,(4(3))
5. 5,(2(1))
6. 1,1,9
7. 1,5,5
8. 7,(2(1))
9. 5,(4(1))
10. 1,(4(5))

Lesson 21: Logic Numbers

1-6-2-10-3-7-4-9-5-8

Lesson 22: Checkerboard Paradox

A close look at the slanted sides of the trapezoidal and triangular pieces shows that they can't be perfectly aligned, and the diagonals of the two smaller rectangles of the 5 x 13 grid are 2 x 5 and 3 x 8, making different slopes for each line. But since the difference is so small (we're talking the difference of 0.025: 2/5 versus 3/8) your eye can be fooled into thinking that the slope of the lines are equal and the grid matches up.

Lesson 24: Magic Squares

1. 99
2. 16
3. 66
4. 89
5. 34
6. 5
7. See magic square below:

2	7	6
9	5	1
4	3	8

8. 1
9. 5
10. 14

Lesson 25: Bagels

1. Wrong guess
2. One of the digits is right but in the wrong position
3. One of the digits is right and in the correct position
4. 76
5. 672
6. 75
7. 25
8. 36
9. 75
10. 1375

Lesson 27: Don't make a triangle

1. Is a three-sided figure
2. Hexagon
3. 2
4. 1
5. 2
6. 3
7. The player would have made one triangle with her own ink which is against the rules of the game
8. No mistake made
9. The player would have made one triangle with her own ink which is against the rules of the game
10. Quadrilateral

Lesson 28: Math dice Game

1. (6 x 3) + 2 = 20
2. (6 x 5 x 4) + (4 x 3) = 96
3. (4 x 4 x 2) + 1 +1 + 6 = 40
4. (4 x 2 x 3) + 1+1 +1 = 27
5. (6 x6) + (5 x 5) + 2 = 63
6. 4 + 5+ 1+ (2 x 2 x 2) = 18
7. 4 x (6 + 5) = 44
8. (6 x 5 x 8) –(4 x 3 x 3) = 84
9. (2 x 2 x 3) + (3 x 4) = 24
10. 3 x (6 +5 + 3 + 3 + 1) = 54

Lesson 29: Big Numbers

1. 3,600
2. 86,400
3. 365
4. A number should be between 1,000,000,050 and 1,000,000,000
5. A number should be between 1,000,000,000 and 999,999,950
6. 1,000,000,000,000
7. 1,000,000,000
8. 10,000
9. 99,999,999,999

Lesson 30: Exponents

1. 1,000,000,000
2. 1,000,000,000,000
3. 10,000,000
4. 100,000,000,000,
5. 10^9
6. 10^{12}
7. 10^7
8. 10^{11}
9. 10
10. 2

Lesson 31: Math at Rock Concert

1. 1100feats/sec
2. 186,000miles/sec
3. Distance = speed x time
4. 200m
5. 3100miles/min
6. 18.33feat/min
7. 0.0645sec
8. 0.1818sec
9. 186,000miles
10. 110feats

Lesson 32: Rail Fence Cipher

1. CMTMR OA2MO EOORW TP
2. HHSRI DTTAR OTEAA REATI PR
3. HICRY BGESA RAAXZ
4. TEAEE EOHYR HRNWZ
5. CMAMP AEOET YLCXZ
6. CEMRI YFEOT OONOI MORWM FC
7. LVHBI EEEEN ATMHD
8. WIBHE OELEE SNWLT ROXQZ
9. I AM COMING NOW
10. I AM COMING NOW

Lesson 33: Twisted Path cipher

1. 15
2. 30
3. 25
4. 20
5. Spiral in clockwise direction from the center
6. Spiral in anticlockwise direction from the first cell
7. AVURO EWILL EKYAD OTFLI
8. AYEHT WMONM TSXEN ECOYA
9. HENNE TTTEN OIYAD GNPDA IWILL
10. ETHTA ERBIW ILLEZ

Lesson 34: Shift Cipher

1. KPNOY DKKHS WOQDA XAOQZ
2. ESWOQ DAXAO QEJIW QDAIW QEYOZ
3. EWIQD NAABA AAQOQ WHHXZ

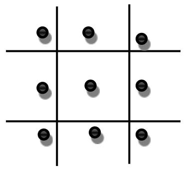
4. KPNOYD KKHDW OCKKZ QAWYD ANOXZ
5. YKIAQ KIUNA OMPAZ
6. WE ARE LEAVING
7. WHAT IS THE QUESTION
8. PLEASE COME TODAY
9. I AM ON MY WAY
10. SHE IS A GOOD SINGER

Lesson 35: Date Shift Cipher

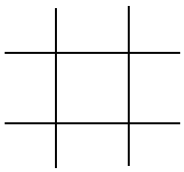
1. Numerical equivalent of the date
2. Backward
3. Polyalphabetic
4. 040498
5. 012812
6. 113011
7. CRNCY XMBIU VNZQX
8. TNEZH BVKJV OULKF UZQXZ
9. GO AT TEN AM
10. MEET ME AT SIX PM

Lesson 36: Pig pen Cipher

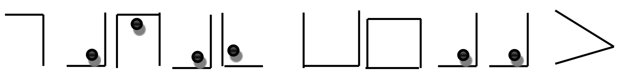
1. The second pig pen is composed of a cross while the fourth one is composed of the cross with dots.
2. A



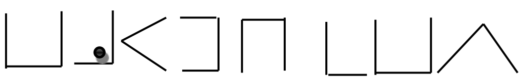
3.



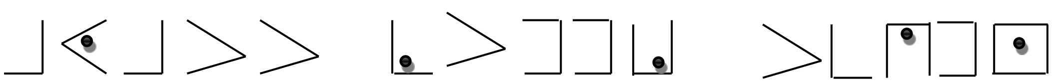
4.



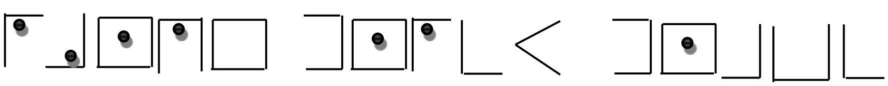
5.



6.



7.



8. People came
9. He came in peace

Lesson 37: Polybius Cipher

1.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y/Z

2. 11;35 and 13 respectively
3. 32-15-45-51-44-32-15-11-52-15
4. 24-11-33-32-15-11-52-24-34-22
5. 45-23-55(1)-53-24-32-32-13-35-33-15
6. 44-23-15-24-44-23-15-43-15
7. LOOK AT THE TABLE
8. IT IS HIDDEN
9. HERE IS THE PAPER
10. LOOK AT MY BOOK

Lesson 38: Cracking Ciphers

1. Is the idea of getting the original message from the cipher without using the key
2. Cracking implies getting the original message of the cipher without the key while with decoding, the key is used
3. The person has to be familiar with the common letters and words used in English
4. E
5. T
6. I,A
7. OF,TO
8. THE,AND
9. THAT

Lesson 39: Playfair Cipher

1. The key
2. B5
3. KH
4. HG
5. 56
6. SP YB JZ GA DK PD

7. XS DK YB 73 X6
8. THAT IS MINE
9. I SAW HER TODAY
10. So as to enable one to apply the three rules when encoding the message

Lesson 40: Telephone Cipher

1. 2 6 6 3 2 2 2 5
2. 5 3 2 8 3 86 3 2 9
3. 4 3 2 7 7 4 8 3 3
4. 8 4 3 9 9 3 6 8
5. 4 3 4 7 2 6 6 4 6 4
6. 7 3 8 3 2 5
7. 4 6 2 4 3 2 3
8. AMERICA
9. UNITED KINGDOM
10. AUSTRALIA

Lesson 41: Scytale

1. Scytale
2. Cylindrical in shape
3. A strip of paper rolled on its (Scytale)curved surface
4. Paper
5. Spartans
6. A Scytale of the same size as the encoder
7. No, The encoder may chose to begin writing the first row of the message some centimeters away then end up beginning another row that the beginning of the paper strip.

Lesson 42: Secret Phonetic code

1. 956
2. 2
3. 6527100
4. 103841
5. 1660215263
6. 995563217
7. The phrase formed should have chmtmrsc or their equivalent
8. The phrase formed should have tsgd or their equivalent

Lesson 43: Factorials!

1. 720
2. 30
3. 5040
4. 24
5. 144
6. 120
7. 0
8. 48
9. 12
10. 240

Lesson 44: Probability

1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. 1
4. $\frac{1}{2}$
5. $\frac{1}{6}$
6. $\frac{1}{6}$
7. $\frac{1}{3}$
8. $\frac{1}{36}$
9. $\frac{1}{18}$
10. $\frac{1}{6}$